SYSTEM IDENTIFICATION AND DAMAGE DETECTION USING WAVELET ANALYSIS: APPLICATIONS IN FRAME STRUCTURES

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Abstract

A novel wavelet analysis is presented in this paper in order to locate damage sources in civil structures. The presence and location of damage are detected by solely analyzing the acceleration time history responses obtained from the structure of interest. Two structures running in various operational environments are investigated: FE – model of a three storey shear-resisting frame excited by band-limited white noise ground acceleration and a prestressed reinforced concrete test beam under impact loading conditions. The presented technique requires the availability of measured data from a previous “reference” or “undamaged” structural state. Using the Haar mother wavelet a first level fast wavelet decomposition of both the reference and actual acceleration measurements is applied, where the approximation and detail coefficients are obtained. Subsequently reconstruction of the actual structural response is performed by combination of the “reference” approximation coefficients and the “actual” detail coefficients. Then, the error, which is the difference between the “reconstructed” signal and the actual acceleration measurement, is defined as the damage-sensitive parameter. The basic idea is that the detail coefficients carry the whole information of discontinuities in the structural time history response at the damaged sites. Therefore, the approximation coefficients previously obtained using the undamaged measured data combined with the actual detail coefficients would not be able to reproduce the newly obtained time history response of the damaged structure. Furthermore, the increase in error would be maximized at sensors instrumented near the actual damage sources.

1 Introduction

The design of civil engineering structures is characterized by two features: carrying capacity and serviceability. However, the buildings undergo various environmental and loading influences during their service life, which can cause a significant damage accumulation. Consequently the structural carrying capacity and serviceability are enormously affected. Therefore the need of reliable non-destructive evaluation techniques and detection of damages at the earliest possible stage has been pervasive throughout the civil engineering community at the last decade. As reported by Sohn et al. [1] the process of implementing damage detection strategies can be referred to as structural health monitoring. Vibration-based health monitoring techniques, also “global” monitoring methods [2], rely on the fact that damage causes changes in the local structural damping (energy dissipation) and stiffness, and therefore in the global dynamic properties of the structure. Several tools for a continuous safety vibration-based monitoring during the structural service life are reported by Wenzel et al. [3] and [4]. To identify the presence of damage in a mechanical system, often the frequencies and other modal parameters are calculated only from measurements of the dynamic struc-
tural response without knowing the input loading force, e.g. *Operational Modal Analysis*. The process of damage identification is more than detection of changes in the dynamic structural characteristics. Based on the work of [5], four levels of structural health monitoring and damage identification are discriminated: (1) **detection** of damage presence in a structure, (2) **localization** of damage source, (3) **quantification** of damage severity, and (4) **prediction** of the remaining structural service life. An ideal structural health monitoring system should be capable of providing cost-effective and reliable damage identification.

There are several methods operating either in the frequency or in the time domain. Consequently, often useful information about structural changes through the unused domain is discounted. Doebling et al. [6] referred that presence of damage is a local phenomenon which tends to be captured by higher frequency modes. However, this fact adds difficulties to the implementation of damage identification through the frequency domain by the classical Fourier analysis. To overcome these deficiencies, the application of time-scale or time-frequency analysis is required. Staszewski et al. [7] used the Wigner-Ville time-frequency distribution to detect source of damage in a gearbox. Another very useful time-frequency transform that inspire nowadays the attention of researchers is the wavelet transform (WT).

The authors present in this paper the structural health monitoring of frame structures in the context of a wavelet-based analysis. Two different systems are investigated: a numerically modelled (FEM) shear resisting steel frame with three storeys excited by an artificial generated 20 Hz band-limited white noise ground motion. Three damage scenarios are investigated on this structure, namely plastic hinge simulation underneath each storey respectively. The second mechanical system of interest is a prestressed reinforced concrete test beam under impact loading conditions. Seven structural states are observed, where the tendons are released successively (prestressing forces reduction).

The applied damage identification technique can be described as a five-part process: (1) operational evaluation, (2) data acquisition and standardization, (3) first level of wavelet decomposition, (4) data reconstruction, and (5) damage parameter development. The presented approach requires the availability of acceleration records from at least two structural states. The method is based only on the analysis of measured vibration data, making this technique very attractive for its implementation into automated health monitoring and decision support systems.

## 2 Fast Wavelet Transformation

The wavelet transformation of a signal $x(t)$ is a time-scale decomposition obtained by stretching and translating along the time axis a chosen basis function (*mother wavelet*). Thus, the one-dimensional wavelet transformation projects the signal into a two-dimensional space:

$$W_x^\psi(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^\ast\left(\frac{t-b}{a}\right) dt ,$$  

where $b$ is the parameter localising the wavelet function in the time domain, $a$ is the dilation parameter defining the analysis window stretching, and $\psi^\ast$ is the complex conjugate of the mother wavelet function. This basic function is used to generate a family of wavelet functions as follows:

$$\psi^{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right).$$  

The process of wavelet analysis is represented in general by the wavelet transform in continuous or discrete version. For the continuous wavelet transform, the basis function can always be repre-
presented by an analytical function. For most of the applicable wavelets, however, the projection of
the signal \( x(t) \) into the continuous time-scale domain requires a considerable number of numerical
operations. Thus, a discrete representation of the wavelet transformation is available, where the pa-
rameters \( a \) and \( b \) become discrete values:

\[
W_{m,n} = \sum_{m \geq 1} \sum_{n \in \mathbb{Z}} x(t) \psi_{m,n}(t),
\]

where the wavelet \( \psi_{m,n}(t) \) is expressed as follows:

\[
\psi_{m,n}(t) = 2^{-m} \psi(2^{-m}t - n).
\]

The wavelet function in this paper is the well-known Haar wavelet (A. Haar, 1910) as shown in
Figure 1. The origin is the following function

\[
\psi(t) = \begin{cases} 
1, & 0 \leq t \leq 0.5; \\
-1, & 0.5 < t \leq 1; \\
0, & \text{else.}
\end{cases}
\]

Note, that the functions \( \{ \psi_{m,n} | m,n \in \mathbb{Z} \} \) is an orthonormal base of \( L^2(\mathbb{R}) \). Considering
the stepwise approximation of the function \( x(t) \) (Bàni [8]), where \( x(t) \in L^2(\mathbb{R}) \) and \( m \in \mathbb{Z} \),
with the constant step width \( 2^m \) leads to the constant value

\[
x_{m,n} = 2^{-m} \int_{2^m} x(t) dt.
\]

This expression represents the mean value of the function \( x(t) \) on the interval \([n2^m, (n+1)2^m] \). Furthermore, it can be represented that this is also the “mean value of the mean values” on the both
equal subintervals \([2n2^{m-1}, (2n+1)2^{m-1}] \) and \([(2n+1)2^{m-1}, (2n+2)2^{m-1}] \), namely:

\[
x_{m,n} = \frac{x_{n-1,2n} + x_{n-1,n+1}}{2}.
\]

Using the fact that the amplitude \( \psi_{m,n}(t) \) is equal to \( 2^n \) gives for the wavelet coefficients

\[
V_{m,n} = 2^n \frac{x_{n-1,2n} - x_{n-1,n+1}}{2}.
\]

Using the Haar scale function (see Figure 1), where its scale coefficients \( u_{m,n} \) are defined as

\[
u_{m,n} = 2^n x_{m,n} = 2^n \int_{2^m} x(t) dt,
\]

the stepwise approximation of the function \( x(t) \) can be represented by the following relationship:
Thus, modifying equations (7) and (8) by expression (10) leads to basic system equations of the fast Haar wavelet transformation:

\[ u_{n+1,m} = \frac{u_{n,2m} + u_{n,2m+1}}{\sqrt{2}}, \]
\[ v_{n+1,m} = \frac{v_{n,2m} - v_{n,2m+1}}{\sqrt{2}}. \]

Inverting equation (11) gives the scale coefficients \( u_{n,m} \):

\[ u_{n-1,2m} = \frac{u_{n,m} + v_{n,m}}{\sqrt{2}}, \]
\[ u_{n-1,2m+1} = \frac{u_{n,m} - v_{n,m}}{\sqrt{2}}, \]

which allows to synthesize the stepwise approximation of the function \( x(t) \). The transition from \( u_n \) to \( u_{n+1} \) and to \( v_{n+1} \) with respect to equation (11) can be represented by following relationships:

\[ u_{n+1} = (\downarrow 2) A(u_n), \]
\[ v_{n+1} = (\downarrow 2) D(v_n), \]

where \( A(u_n) \) means the “moving average”, \( D(v_n) \) is the “moving difference” and \( (\downarrow 2) \) stands for the “downsampling”.

Equation (13) is also known as the “analysis bank” of the so called “Haar filter bank” shown in Figure 2. The right part of the filter bank represents the “synthesis bank”, where \( (\uparrow 2) \) indicates the “upsampling”. This filter bank is a “perfect reconstruction” filter bank, which means that every signal can exactly be reconstructed, if the coefficients \( u \) and \( v \) are not modified. Note, that Figure 2 represents only the first level of Haar fast wavelet decomposition.

3 Damage Detection Procedure

The damage detection technique presented in this paper requires the existence of at least two monitored structural states. In general damage causes changes in the system vibration behaviour particularly with regard to the energy dissipation (damping) and the structural stiffness. Outgoing from an “undamaged” also “reference” structural state, the method consider the mechanical system of interest at a subsequent time instant. This supposes similar structural conditions to eliminate any serious influence of inconsistent boundary conditions, whereas the operational conditions can vary. Additionally an identical sensor layout is required in order to locate any detected system changes.

First, all discrete acceleration signals should be made comparable to each other. For this purpose they are standardized:
\[
\hat{x}(t) = \frac{x(t) - \mu_x(t)}{\sigma_x},
\]

where \( \hat{x}(t) \) is the standardized time signal, \( \mu_x(t) \) is the mean of \( x(t) \), and \( \sigma_x \) is its standard deviation. However, for simplicity, \( x(t) \) is used to denote \( \hat{x}(t) \) hereafter.

For each time series \( x(t) \) of all structural states considered, a first level of Haar wavelet decomposition is applied as mentioned above. This procedure provides two vectors per analysed acceleration signal, namely the approximations \( u \) and the details \( v \) of \( x(t) \) (see Figure 2). The approximations are the high-scale, low-frequency components, whereas the details represent the low-scale, high-frequency details of the signal. However, the presence of damage as a local incident is generally captured by the higher frequency modes, as reported by Doebling et al. [6]. Thus, information about any system changes especially due to damage shall be stored in the details vector \( v \).

Next, a simulation of the actual “damaged” structural state is performed by wavelet reconstruction. Using the Haar synthesis filter bank, the “reference” approximation coefficients are combined with the “actual” detail coefficients. In other words, the obtained damage information is superimposed on the approximation information for the “healthy” time history response.

Finally, synthesis error \( \varepsilon_{\text{synthesis}} \) for the actual structural state can be obtained as follows:

\[
\varepsilon_{\text{synthesis}} = x(t)_{\text{synthesis}} - x(t)_{\text{actual}}.
\]

When the reconstructed system response is not a good representation of the newly obtained time signal, there would be a significant change in standard deviation of \( \varepsilon_{\text{synthesis}} \), \( \sigma(\varepsilon_{\text{synthesis}}) \). This consideration is simply based on the definition for standard deviation, i.e. measure of the degree of dispersion of a data from its mean value.

4 Examples

The presented approach is verified by means of two mechanical systems working in various operational environments.

4.1 Steel Frame excited by White Noise Ground Motion

A three storey shear resisting steel frame is investigated by means of the finite element method. The system properties are depicted in Figure 3a. Presence of damage is simulated by means of plastic hinges. For this purpose cross section reduction underneath of the story of interest is applied. In other words, three damage scenarios are modelled. An 20 Hz band-limited artificial generated white noise is used as ground motion (see Figure 3b).

Figure 3: Frame Structure and Ground Excitation
Applying the damage identification approach by means of first level of Haar wavelet decomposition and reconstruction, and subsequently consideration of the standard deviation of the synthesis error $e_{\text{synthesis}}$ leads to results shown in Figure 4. It can be seen, that the largest increase in the damage parameter is obtained at the nearest measurement point to the damage site.

4.2 Prestressed Reinforced Concrete Beam under Impact Load Conditions

Next, a real structure is investigated: reinforced concrete beam prestessed by six tendons (see Figure 5). The influence of the prestressing forces on the global system behaviour is tested. In other words, starting form a completely prestressed state, the tendons are released successively. The final state is characterized by absence of any prestressing forces. The vibration behaviour is observed under impact load conditions (impact load of 120.0 kg on the right free end), where structure is instrumented by eight sensors positioned on the upper surface. Note, the impact load history of each structural state differs from those of the other prestressing cases. Using the obtained acceleration time histories the concrete cracks are detected and localized by means of the wavelet approach presented above. The standard deviation is again as damage parameter specified, but from the square of the synthesis error $e_{\text{synthesis}}^2$ at each prestressing state. Note, the comp-
pletely prestressed structure represents the “reference” state in this analysis.

The obtained damage distributions over the beam longitudinal axis are depicted in Figure 6. As expected, the main presence of concrete cracks is located at the supports. However, because of the impact load the damage parameter values at the right support are greater than those at the left one. Additionally, the more tendons are released the more damage at the middle of the span can be observed. Note, that the significance of the damage parameter is not influenced from the inconsistence of the impact load at each investigated structural state.

5 Conclusions and Outlook on Future Work

This paper presents a wavelet approach for damage identification in civil frame structures. The technique is based on the first level fast Haar wavelet decomposition. Combining the approximation coefficients from a “reference” structural state and the detail coefficients from the actual “damaged” state the vibration response is reconstructed. The standard deviation of the error between the measured and predicted acceleration time signal is chosen as a damage-sensitive parameter. Finally, in order to verify the procedure, two civil structures working in various operational environments are observed.

The presence of damage and the damage sources can be detected by means of the presented technique. The method requires the availability of measured acceleration data of at least two structural states, where the first one is assumed as an undamaged “reference” state. Additionally similar structural conditions are supposed, in order to eliminate the influence of inconsistent boundary conditions. A significant advantage of the applied procedure is its robustness against varying loading conditions. The method is based only on the analysis of acceleration time history records, making this approach very useful for its implementation into automated structural health monitoring and decision support systems.

Future work will be devoted to the extension of the presented approach to quantification of damage and to prediction of remaining structural life. Furthermore, more complex mechanical systems and a wide range of operational and environmental conditions shall be investigated. The analysis of different damage scenarios, especially non-linear damage phenomena, will also be part of future activities of the authors.

6 References


