OPERATIONAL MODAL ANALYSIS
AND
ASSESSMENT OF HISTORICAL STRUCTURES

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Abstract
The paper summarizes the results of recent ambient vibration tests and operational modal analyses carried out on three historical structures: the masonry bell-tower of the Monza cathedral (1605), the Morca suspension bridge (1928) and the Carpineto cable-stayed bridge (1977, the last of Morandi’s cable-stayed bridges).

1 Introduction
The experimental modal analysis of historical structures has become of increasing concern since often the unique structural styles of these systems as well as the uncertainties about geometry and materials added significant difficulties in both accurate structural analysis and assessment of "as built" behaviour. The knowledge of the modal properties of historical structures is highly desirable either for health monitoring or to better understand the structural performance during extreme load conditions such as those caused by seismic events.

The experimental modal analysis of historical structures is generally carried out by using output-only measured data, since artificial excitation often exhibits a problem while the environmental loads are always present and hence the test implies a minimum interference with the normal use of the structure. The identified modal parameters, in turn, can be systematically correlated with the results of finite elements analyses to validate and to improve theoretical models or the dynamic signature can be directly used to evaluate the changes in the modal parameters and the correlation with structural modifications or damage.

This paper summarizes the results of some recent investigations carried out on different historical structures:

1. the masonry bell-tower of the Monza cathedral, 74 m high, dating back to the XVII century (Refs. [4]-[5], [11]);
2. the Morca suspension bridge with timber deck, about 92 m long, erected in 1928 (Ref. [16]) and recently retrofitted by replacing the original chains;
3. the Carpineto cable-stayed bridge, with pre-stressed concrete stays, designed by R. Morandi in the 70’s (Ref. [14]).

In each investigation, the operational modal analysis was carried out by using well-known procedures, in the frequency domain (Refs. [2]-[3], [6]-[7]) and/or in the time domain (Refs. [2], [15], [17]), with different aims including the f.e. model-based assessment of damage, the direct use of the modal signature to investigate the evolution of the structural conditions and the validation of analytical models to be used for a seismic analysis as a part of seismic risk assessment.
2 The bell-tower of the Monza cathedral

2.1 Description of the damage and on-site investigations:

The bell-tower of the Monza cathedral (Fig. 1) was built between 1592 and 1605. The load-bearing walls of the tower, 74 m high and 1.40 m thick, were made with solid masonry bricks and showed passing-through, large and potentially dangerous vertical cracks especially on the West and East sides. Other vertical and very thin cracks can be observed mainly on the inner faces of the bearing walls. The observed crack pattern is present approximately from a height of 11.0 m to 23.0 m.

In order to assess the structural condition of the tower, an extensive investigation was carried out on-site using non-destructive and slightly destructive techniques together with laboratory tests and analytical calculations. The complete results of on-site investigations are reported in Refs. [4]-[5]. The investigation included an accurate geometric survey of the structure and of the crack patterns and distribution. Flat-jack tests were performed in selected points to directly estimate the stress level caused by the dead load and to check the stress-strain behaviour of the masonry under compression; specifically, the Young's modulus was generally ranging between 985 and 1380 N/mm² while a Poisson ratio of 0.07–0.20 was detected. In addition, a first series of dynamic tests using four servo-accelerometers was carried out in 1995 to evaluate the natural frequencies of the tower (Ref. [4]).

2.2 Ambient vibration tests and operational modal analysis:

Extensive full-scale dynamic tests were carried out at the beginning of July 2001 and the dynamic response of the Tower was measured at 20 different locations, with the excitation being associated to environmental loads and to the bell ringing. Fig. 2 shows a schematic representation of the sensor layout. WR-731A piezoelectric sensors were used during the tests; these sensors allowed acceleration or velocity responses to be recorded. Ambient vibration response (in terms of both acceleration and velocity) was acquired in 2280 s records per channel at a sample rate of 200 Hz. Due to the low level of ambient excitation that existed during the tests, the maximum recorded velocity ranges up to about 0.15 mm/s.

5 vibration modes were identified in the frequency range of 0–8 Hz using two different output-only

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode Type</th>
<th>( f_{PP} ) (Hz)</th>
<th>( f_{FD} ) (Hz)</th>
<th>( D_F ) (%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bending mode in E–W / N–S direction</td>
<td>0.586</td>
<td>0.598</td>
<td>2.05</td>
<td>0.9984</td>
</tr>
<tr>
<td>2</td>
<td>Bending mode in N–S / E–W direction</td>
<td>0.708</td>
<td>0.708</td>
<td>0.00</td>
<td>0.9989</td>
</tr>
<tr>
<td>3</td>
<td>Torsional mode</td>
<td>2.456</td>
<td>2.417</td>
<td>1.59</td>
<td>0.9929</td>
</tr>
<tr>
<td>4</td>
<td>Bending mode in E–W / N–S direction</td>
<td>2.731</td>
<td>2.722</td>
<td>0.33</td>
<td>0.9597</td>
</tr>
<tr>
<td>5</td>
<td>Bending mode in E–W / N–S direction</td>
<td>5.706</td>
<td>5.713</td>
<td>0.12</td>
<td>0.9923</td>
</tr>
</tbody>
</table>
techniques: the Peak Picking method (PP, see e.g. Ref. [3]) and the Frequency Domain Decomposition (FDD, Ref. [6]).

Table 1 summarises the identified modes and their classification. Furthermore, Table 1 compares the corresponding mode shapes and scaled modal vectors obtained from the PP and the FDD techniques through the frequency discrepancy $D_F = \frac{|f_{PP} - f_{FDD}|}{f_{PP}}$ and the MAC (Ref. [1]). Inspection of the correlation values listed in Table 1 clearly shows a very good agreement between the PP and the FDD techniques in terms of both resonant frequencies (with the maximum differences not exceeding 2.05%) and modal deflections (with minimum MAC value of 0.9597).

Finally, the experimentally determined bending modes of the tower are shown in Fig. 3. It should be noticed that, notwithstanding the nearly symmetric shape of the tower, the dominant bending modes of the system generally show coupled motion in the two main E–W and N–S directions: the experimental modal analysis suggests either a strong coupling between the tower and the cathedral or a non-symmetric stiffness distribution (as the one which has to be expected basing on the crack distribution). Hence, this hypothesis was investigated using a 3D finite element model.

2.3 Finite element modelling and updating:

A finite element model of the bell-tower was created based on the geometric survey. The tower was modelled by using 8-node brick elements while the dome was represented by 4-node shell elements. The model consisted of 4944 nodes, 3387 solid elements and 80 shell elements with 14286 active degrees of freedom (Fig. 4). In formulating the model, the following hypotheses were adopted: (a) the tower footing was considered as fixed; (b) a weight per unit volume of 18.0 kN/m$^3$ was assumed for the masonry; (c) the Poisson’s ratio of the masonry was held constant and equal to 0.15; (d) the connection between the Southern wall of the bell-tower and the facade of the cathedral was accounted for by introducing rigid constraints normally to the wall; in the orthogonal direction
the interaction between the tower and the cathedral was simulated by an uniform distribution of linear elastic springs of constant $k$ (Fig. 4). The range of variation of $k$ was estimated in order to ensure a broad correspondence between theoretical and experimental mode shapes.

A non-homogeneous distribution of the Young’s modulus was considered in order to adequately represent the damaged areas of the tower; specifically, six different values of the elastic modulus were considered in the model. Since the major cracks were placed along the East- and West-sides of the building up to about 20.0–23.0 m, the lower part of the tower (up to 23.0 m) was divided into five sub-regions by distinguishing the four sides and the corner properties, as it is shown in Fig. 5; a further elastic modulus was introduced to represent the average behaviour of the masonry in the upper part of the tower. Hence, the finite element model updating was carried out with respect to the set of seven structural parameters summarized in Table 2:

1. the Young’s moduli $E_i$ ($i=1,2,...,5$) in the lower part of the tower;
2. the Young’s modulus $E_6$ in the upper part of the tower;
3. the elastic constants $k$ of the springs placed along the contact area between the Cathedral and the bell-tower in the direction of the Southern wall.

The uncertain structural parameters were estimated by minimising the difference between theoretical and experimental natural frequencies through two different well-known identification algorithms: the Douglas-Reid (DR) procedure (Ref. [9]) and the inverse eigensensitivity (IE) method (Ref. [8]), with both techniques requiring the estimate of upper and lower bounds for the updating parameters.

Table 2 summarises the optimal estimates of the structural parameters obtained from DR and IE procedures, the base value of the parameters and the assumed lower and upper limits. The inspection of Table 2 first reveals coherent information on the stiffness distribution. Furthermore, both the estimated sets of parameters seem to represent quite well the damage distribution of the tower and are also in good agreement with the double flat-jack results. Specifically, a low stiffness ratio is detected for the Eastern and Western load-bearing walls up 23.0 m, with the elastic modulus in such regions being about the half of the values obtained in the other parts of the tower. Furthermore, in the lower part of the tower the Young’s modulus turns out to be higher in the corner zones than elsewhere; the traditional construction, in fact, is generally characterised by a more accurate building technique in the corners.

Fig. 6 shows the vibration modes of the DR updated model corresponding to the experimental ones and the correlation with the experimental modal behaviour. The natural frequencies of the DR
updated model are practically equal to the experimental ones, as it has to be expected since the model updating was based on the minimisation of frequency discrepancies. The correlation between mode shapes shows very good agreement with the experimental results for the first two modes (with the $MAC$ being greater than 0.97); for the higher modes, the $MAC$ is in the range 0.86–0.87 so that appreciable average differences are detected. Such differences are probably to be related either to the simplified distribution of the model elastic properties (which were held constant for large zones of the structure) or to a relative lack of accuracy in the experimental evaluation of the higher mode shapes. A similar correlation was found from $IE$ updated model as well.

$FEM = 0.585$ Hz  $FEM = 0.709$ Hz  $FEM = 2.455$ Hz  $FEM = 2.726$ Hz  $FEM = 5.698$ Hz
$P = 0.586$ Hz  $P = 0.708$ Hz  $P = 2.456$ Hz  $P = 2.731$ Hz  $P = 5.706$ Hz

$MAC = 0.9874$  $MAC = 0.9745$  $MAC = 0.8614$  $MAC = 0.8602$  $MAC = 0.8721$

Figure 6. Vibration modes of the updated model (Douglas-Reid method)

Table 2. Structural parameters of the updated models

<table>
<thead>
<tr>
<th>Structural Parameter</th>
<th>Lower Value</th>
<th>Base Value</th>
<th>Upper Value</th>
<th>DR</th>
<th>IE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (N/mm$^2$)</td>
<td>800</td>
<td>1400</td>
<td>2500</td>
<td>1718</td>
<td>1772</td>
</tr>
<tr>
<td>Corner (height ≤ 23 m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_2$ (N/mm$^2$)</td>
<td>500</td>
<td>1400</td>
<td>1800</td>
<td>930</td>
<td>751</td>
</tr>
<tr>
<td>West wall (height ≤ 23 m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_3$ (N/mm$^2$)</td>
<td>800</td>
<td>1400</td>
<td>2500</td>
<td>1591</td>
<td>2060</td>
</tr>
<tr>
<td>South wall (height ≤ 23 m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_4$ (N/mm$^2$)</td>
<td>500</td>
<td>1400</td>
<td>1800</td>
<td>742</td>
<td>880</td>
</tr>
<tr>
<td>East wall (height ≤ 23 m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_5$ (N/mm$^2$)</td>
<td>800</td>
<td>1400</td>
<td>2500</td>
<td>1493</td>
<td>1440</td>
</tr>
<tr>
<td>North wall (height ≤ 23 m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_6$ (N/mm$^2$)</td>
<td>800</td>
<td>1400</td>
<td>1800</td>
<td>1789</td>
<td>1729</td>
</tr>
<tr>
<td>All walls (height ≥ 23 m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k$ (kN/m$^2$)</td>
<td>34450</td>
<td>68900</td>
<td>86250</td>
<td>69730</td>
<td>67987</td>
</tr>
</tbody>
</table>
3 The Morca suspension bridge

3.1 Description of the bridge and historic background:

The Morca bridge (Figs. 7-9) spans the Sesia river and is placed in the neighborhood of Varallo Sesia (a town about 20 km far from Vercelli, Italy). The bridge, originally constructed in 1928, belongs to the traditional and cultural heritage of the region, where 9 suspension bridges have been constructed between 1843 and 1928 (Ref. [16]), with 3 of these bridges still standing today.

Fig. 7 shows an elevation drawing of the bridge, based on the recent inspection and field survey. Fig. 8 shows a photograph of the bridge taken in the current configuration. The suspended deck is 91.6 m long and there are no side-spans; the deck is in solid timber boards supported by a grid of timber stringers and floor beams (Fig. 9). The floor system is completed by two lateral stiffening trusses. The bottom chords of the truss are the outer stringers of the deck; each panel of the truss is stiffened by X-bracings and the top chord is used as handrail. The deck was originally supported by two couples of cables, each composed of a spiral strand with 61 galvanized steel wires. Steel hanger rods, of 20 mm diameter and generally spaced every 1.60 m, extend from the cables to the floor beams. The hanger rods are canted outward from the deck to the suspension cables and effectively pinned at the ends. The masonry towers up-rise the floor system of about 10.50 m.

The bridge underwent during its history to minor modifications consisting in the replacement of the damaged timber elements; furthermore, in 2003 a more important retrofit intervention was planned and carried out. This intervention basically involved the complete replacement of the suspension system (currently consisting of two couples of open spiral steel cables as in the original arrangement), the stiffening of the floor system carried out by adding two couples of steel stringers (Fig. 11) and the replacement of the damaged timber elements.

After the retrofit, the bridge was opened again to pedestrians and light vehicular traffic, with the maximum allowed weight of the vehicles being limited to 35 kN at 10 km/h speed. Furthermore, a research program was activated to assess the actual structural behaviour of the bridge and to assist in its future preservation.

The first part of the investigation, described in Ref. [12], consisted in the dynamic-based assessment of the structure and involved field survey of the actual deformed configuration due to dead load, ambient vibration testing and experimental modal analysis, analytical modelling and model updating; in addition to dynamic tests, the vertical deflections of the bridge under live load were measured to further validate the theoretical model.

The objective of the second part of the research, herein described, was to repeat the dynamic tests in order to evaluate any changes in the modal parameters and the correlation with structural modifications or damage.
3.2 Ambient vibration tests, operational modal analysis and damage evaluation:

Two series of ambient vibration tests were conducted on the Morca suspension bridge, at the beginning of May 2004 and at the end of July 2004. In both tests, a 16-channel data acquisition...
system with 14 WR-731A piezoelectric accelerometers was used. In the test of May, preliminary dynamic investigations from heel drop, jumping and heavy walking indicated that vertical vibrations, either bending or torsion, were readily excited while lateral vibrations could only be excited with great difficulty. Hence, it was decided to measure only the vertical accelerations, with the sensor layout shown in Fig. 10. In the test of July, the same layout of vertical sensors was adopted but few sensors were placed in the transverse direction as well. In both tests, the ambient acceleration time-histories were recorded for 3000 s at a sample rate of 200 Hz.

The operational modal analysis was carried out using the Enhanced Frequency Domain Decomposition (EFDD, Ref. [7]) and the Stochastic Subspace Identification techniques (SSI, Refs. [15], [17]; these techniques are available in the commercial program ARTeMIS (Ref. [2]).

Table 3. Comparison between the modal parameters estimated in the two tests

<table>
<thead>
<tr>
<th>Mode</th>
<th>EFDD</th>
<th>SSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(f_{\text{MAY}}) (Hz)</td>
<td>(f_{\text{JULY}}) (Hz)</td>
</tr>
<tr>
<td>V1+</td>
<td>0.443</td>
<td>0.449</td>
</tr>
<tr>
<td>V2+</td>
<td>0.646</td>
<td>0.653</td>
</tr>
<tr>
<td>V1−</td>
<td>0.738</td>
<td>0.745</td>
</tr>
<tr>
<td>V2−</td>
<td>0.965</td>
<td>0.989</td>
</tr>
<tr>
<td>V3+</td>
<td>1.264</td>
<td>1.382</td>
</tr>
</tbody>
</table>

In both tests, 3 bending modes and 2 torsion modes were identified in the frequency range 0–1.5 Hz, with significant differences in the mode shapes being detected between the two tests, especially on the Varallo side of the bridge. The differences are evident simply by inspecting the mode shapes, as it is shown in Fig. 11 where the mode shapes estimated via EFDD in the tests of May and July are super-imposed for a selected number of mode shapes. Table 3 compares the corresponding mode shapes and scaled modal vectors estimated in the two different tests, by means of the EFDD and SSI techniques, through the frequency discrepancy \(D_F = (f_{\text{JULY}} - f_{\text{MAY}})/f_{\text{MAY}}\) and the MAC.

In order to better understand the origin of the detected differences:

1. the modal displacements were interpolated via cubic-spline technique and the COMAC (Ref. [13]) values were evaluated corresponding to each transverse floor beam. The plot of \([1 - \text{COMAC}]\) along the deck is shown in Fig. 12 and clearly highlights the locations where the major differences in the mode shapes take place;
2. Accurate field inspection was carried out and the collapse of the bolt connecting the vertical timber struts of the stiffening trusses and the deck stringers was detected basically in the same zones where \([I\text{-COMAC}]\) peaks. The position of the damaged connections is schematically shown in Fig. 12 to highlights the fairly good correlation between the prediction obtained by the \([I\text{-COMAC}]\) indicator and the actual damage location.

4 The Carpineto cable-stayed bridge

4.1 Description of the bridge:

The Carpineto viaduct (Fig. 13) was designed by R. Morandi and its construction was completed in 1977. The viaduct is part of the freeway between Potenza and Sicignano in central Italy and includes two identical cable-stayed bridges (south-bound and north-bound); the subject of the paper is the south-bound bridge.

The general arrangement of the bridges is shown in Fig. 13; each bridge has a main span of 181.0 m and consists of two box girder cantilevers which are connected by a simply supported drop-in girder. Each cantilever, 65.50 m long, is hinged at the tower footing and supported by a couple of pre-stressed concrete forestays; the transverse beam into which the forestays are anchored is again of box section. The forestays are in turn supported by two inclined A-shaped concrete towers with the pre-stressed concrete backstays being anchored to large gravity blocks.

The main structural characteristic of the bridge is the suspension system, since the stays consist of pre-tensioned steel strands (having 320 parallel wires, each of 12.5 mm diameter) encased in a pre-stressed concrete beam (0.80 m wide by 1.10 m deep); such unusual elements give to the structure an unique appearance and have made worldwide famous the cable-stayed bridges designed by Morandi.

4.2 Experimental modal analysis and finite element model validation:

The response of the bridge was measured at selected points using PCB (model 393) accelerometers, each with a battery power unit. Fig. 14 shows a schematic diagram of the vertical sensor layout. Logistics demanded less extensive instrumentation of the tower and stays on the Salerno side of the bridge. A series of three set-ups was required to cover the measurement points of Fig. 14, with the sensors placed at points 6, 17-18 in Fig. 14 being used for reference measurements. Ambient vibration response was acquired in about 20’ records (per channel and per test configuration) at a sample rate of 250 Hz.

![Figure 13. Elevation of the Carpineto bridge, Italy (dimensions in m)](image-url)
The modal identification was first limited to the frequency range $0 \rightarrow 4$ Hz, since this range includes all the modes dominating the earthquake response of the analytical model. Within the investigated frequency range $0 \rightarrow 4$ Hz, 13 vertical modes were identified (being two of them local modes of the stay-beams) using the PP and the FDD techniques.

The mode shapes and natural frequencies determined in the experimental program were compared to the predictions of a 3D simplified model, created using "as built" drawings of the bridge. A plot of the model, consisting of 236 elements and 239 nodes, is shown in Fig. 15.

The linear modelling was based on the assumption that the bridge vibrates around its dead load state [10]. In the present case, the sag of the forestays was known since it has been determined by an accurate survey in the field whereas the large axial forces in concrete cables, deck and towers were estimated by approximately simulating the main construction phases and accounted for by using a geometric stiffness matrix which is added to the elastic stiffness matrix. Specifically, the estimated average tensile forces ($T$) in each stay-beam of the suspension system are the following:

- Salerno–side, backstays $T = 15250$ kN
- Salerno–side, forestays $T = 16100$ kN
- Potenza–side, backstays $T = 15770$ kN
- Potenza–side, forestays $T = 16100$ kN

It has to be stressed that the effects of the stay-beams sag and the geometric stiffness greatly affect the dynamic behaviour [10]; if such effects are neglected in the finite element model, the resulting theoretical dynamic properties turned out to be completely different from the actual ones.

Furthermore, the model was developed using the following assumptions: (a) 3D beam elements were used to represent the concrete stays and the towers; (b) since the suspended cantilevers are of box section (and hence no significant warping in torsion has to be expected), a single spine of 3D beam elements was used to model the deck; (c) the tower footings were assumed as fixed; (d) the girder deck is hinged to the tower footing; (e) the design data and the tests carried out on concrete cored samples suggested for the Young modulus of the concrete in the cables ($E_C$), the towers ($E_T$) and the deck ($E_D$), the following values:

- $E_C = E_T = 36000$ N/mm$^2$
- $E_D = 40000$ N/mm$^2$

while a Poisson’s ratio of 0.20 was used.

By examining the computed modes of vibration in the frequency range up to 4 Hz, the following comments can be made:

1. Most modes may be classified as vertical or lateral. Within the 30 computed modes, 13 are dominantly vertical (V), 12 dominantly lateral (L) while coupling in the lateral and vertical directions (LV) occurs in 5 modes only;

2. 9 modes involve vertical bending of the deck, with significant participation of the stay-
beams \( V'(C,D) \);

3. 2 modes involve vertical torsion of the deck, with significant participation of the cables \( V'(C,D) \) and coupling in the lateral direction;

4. 8 modes involve lateral bending of the deck, with generally significant participation of the stays and the towers \( L(C,D,T) \);

5. 11 modes are pure cable modes, with 4 being vertical, 4 lateral and 3 coupled in the vertical and lateral directions.

Table 4 summarizes the characteristics of the theoretical modes in the frequency range 0–4 Hz, the comparison between experimental and analytical frequencies and the correlation of corresponding mode shapes, via the MAC. In general, the model exhibits a very good match with the measured behaviour, with the maximum discrepancy being lower than 6% and the MAC being always greater than 0.95. Hence, the model seems to be adequate for a seismic analysis of the bridge as a part of seismic risk assessment.

Table 4. Comparison between theoretical (FEM) and experimental (FDD) modal parameters

<table>
<thead>
<tr>
<th>Mode N.</th>
<th>Mode Shape ( f_{\text{FEM}} ) (Hz)</th>
<th>( f_{\text{FDD}} ) (Hz)</th>
<th>( D_F ) (%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( V_1'(C,D) ) 0.951</td>
<td>0.961</td>
<td>1.04</td>
<td>0.9864</td>
</tr>
<tr>
<td>2</td>
<td>( L_1(C,D,T) ) 0.966</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>( V_2'(C,D) ) 0.998</td>
<td>1.022</td>
<td>2.35</td>
<td>0.9763</td>
</tr>
<tr>
<td>4</td>
<td>( L_1(C) ) 1.100</td>
<td>1.099</td>
<td>0.01</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>( L_2(C) ) 1.165</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>( L_2(C,D,T) ) 1.224</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>( V_3'(C,D) ) 1.395</td>
<td>1.312</td>
<td>6.33</td>
<td>0.9901</td>
</tr>
<tr>
<td>8</td>
<td>( V_4'(C,D) ) 1.563</td>
<td>1.556</td>
<td>0.45</td>
<td>0.9873</td>
</tr>
<tr>
<td>9</td>
<td>( L_3(C,D,T) ) 1.985</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>10</td>
<td>( L_4(C,D,T) ) 2.002</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>11</td>
<td>( V_5'(C,D) ) 2.075</td>
<td>2.075</td>
<td>0.00</td>
<td>0.9525</td>
</tr>
<tr>
<td>12</td>
<td>( V_6'(C,D) ) 2.342</td>
<td>2.350</td>
<td>0.34</td>
<td>0.9834</td>
</tr>
</tbody>
</table>

5 Conclusion

The successful application of dynamic-based assessment and operational modal analysis to three different historical structures has been demonstrated in the paper. The objectives of each investigation were different and included either the F.E. model validation or the direct use of the modal signature to investigate the structural modifications.

6 Acknowledgements

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7 References


