Abstract

Nowadays Acoustic Modal Analysis often plays a significant role in the optimisation process of the acoustic parameters of all kinds of cavities, e.g. the interior of a car, a train, an airplane etc. A problem with the existing experimental Acoustic Modal Analysis techniques is that one has to use volume acceleration sources which interact with the acoustic system. The Operational Acoustic Modal Analysis (OAMA) technique doesn’t suffer from this problem. An important disadvantage of the OAMA approach is that the estimated mode shapes are not correctly scaled. This scaling problem will be tackled in this paper. We will introduce a sensitivity based rescaling technique for the acoustic domain. Local volume changes of the cavity will make a correct estimate of the mode shapes possible. The technique will be validated by some simulations and experiments.

1 Introduction

In acoustics, sound quality is a topic of increasing importance. As a result of this, also acoustic modal analysis (AMA) gains interest. Especially in the transport sector (cars, busses, trains, airplanes, ... ) acoustic modal analysis and vibro-acoustic modal analysis are already well integrated in the product development process [1, 2, 3]. Acoustic modal analysis has some disadvantages which explain why experimental acoustic modal analysis is not yet widely used in a lot of application areas. A first difficulty is the fact that the volumetric acceleration of the sources has to be known. So, to perform an experimental acoustic modal analysis one needs calibrated volumetric acceleration sound sources which are not widely available. A second problem concerning the sources is that they always have a non-negligible size. So they change the volume of the cavity altering the acoustic system [4].

One can conclude that for the domain of acoustics the problems with the sources make it difficult to perform an experimental modal analysis. Over the past years, a modal identification technique that uses only output data has been developed: operational modal analysis (OMA). It typically uses auto- and cross-power spectra in stead of frequency response functions (FRF’s) [5]. The only disadvantage of OMA compared to EMA is that the mode
shapes are not correctly scaled. However, for several applications this is not important. In these cases OMA is an interesting alternative for EMA [6]. Especially in the domain of civil engineering OMA has been welcomed as a powerful system identification tool.

As mentioned before, until recent it was impossible to calculate correctly scaled mode shapes from a structure using OMA. Recently, several rescaling methods have been introduced. A solution for the structural OMA is the sensitivity-based approach [7, 8]. This approach is based on the fact that when one adds known masses to the structure, the natural frequencies shift. By measuring the frequency shift it is possible to calculate the correct scaling factors. The advantage of this approach is that no additional forces have to be applied. This sensitivity-based rescaling method now makes it feasible to calculate all the modal parameters without using known forces. It is obvious that this solves the problem of EMA in the domain of civil engineering [9]. Recently, the technique has been validated on real-life civil structures (e.g. bridges) [10].

If it would be possible to find an equivalent approach for the domain of acoustics, one would not need the volume acceleration sources anymore. This would be very beneficial because then one could estimate all the modal parameters using only microphones and the operational sound sources. Also there would be no influence of the sound sources which one needs with an EMA. In this paper we will show that such an approach exists. The acoustic sensitivity-based rescaling method will be based on local volume changes in the cavity which is easy to apply.

2 Theoretical aspects

2.1 Analogy between acoustic and mechanical systems

Before we focus on the sensitivity based rescaling technique the analogy between acoustic and mechanical systems will be briefly recapitulated. Dynamic undamped mechanical systems can be modelled in the frequency-domain as:

\[ (-\omega^2 \cdot M^s + K^s) \cdot X(\omega) = F(\omega) \]  

with

- \( M^s \) the structural mass matrix
- \( K^s \) the structural stiffness matrix
- \( X(\omega) \) the displacement vector
- \( F(\omega) \) the force vector

For dynamic undamped acoustic systems the equations are:

\[ (-\omega^2 \cdot M^f + K^f) \cdot P(\omega) = \dot{Q}(\omega) \]  

with

- \( M^f \) the acoustic equivalence of the mass matrix
- \( K^f \) the acoustic equivalence of the stiffness matrix
- \( P(\omega) \) the pressure vector
- \( \dot{Q}(\omega) \) the volume acceleration vector

Note that the acoustic equivalence of the structural mass matrix in fact consists of acoustic compliances. This acoustic compliance is defined as the ratio between the volume displace-
ment and the pressure [11].

\[
M^f_{kl} = \begin{cases} 
0 & \text{for } k \neq l \\
\frac{V_k}{\rho \cdot c^2} & \text{for } k = l 
\end{cases}
\]  

(3a)

with

\[V_k\] the volume of element \(k\) of the cavity
\[
\rho \] the density of the medium
\[
c \] the speed of sound  

(3b)

Also the FRF-matrices are comparable. One can prove that the FRF-matrix of an undamped structural system \(H^s(\omega)\) can be calculated as [12]:

\[
H^s_{kl}(\omega) = \sum_{n=1}^{N^s_m} \frac{\Phi^s_{k,n} \cdot \Phi^s_{l,n}}{\bar{m}^s_n \cdot (\omega^s_n \cdot 2 - \omega^2)}
\]  

(4a)

with

\[
\bar{m}^s_n \quad \text{the structural modal mass of mode } n
\]

\[
\Phi^s_{k,n} \quad \text{the structural mode shape value of mode } n \text{ at point } k
\]

\[
\omega^s_n \quad \text{the resonance frequency of mode } n
\]

\[
N^s_m \quad \text{the number of modes}
\]  

(4b)

The acoustic equivalent equation is [13]

\[
H^f_{kl}(\omega) = \sum_{n=1}^{N^f_m} \frac{\Phi^f_{k,n} \cdot \Phi^f_{l,n}}{\bar{m}^f_n \cdot (\omega^f_n \cdot 2 - \omega^2)}
\]  

(5a)

with

\[
\bar{m}^f_n \quad \text{the acoustic equivalence of the modal mass of mode } n
\]

\[
\Phi^f_{k,n} \quad \text{the acoustic mode shape value of mode } n \text{ at point } k
\]

\[
\omega^f_n \quad \text{the resonance frequency of mode } n
\]

\[
N^f_m \quad \text{the number of modes}
\]  

(5b)

If one compares (4) to (5) one can conclude that the same formula’s can be used for the sensitivity analysis if one uses the correct variables. Table 1 shows the equivalent variables.

Note that \(\bar{m}^f_n \cdot \rho \cdot c^2\) is nothing less than the volume matrix of the cavity weighted by the mode shape of mode \(n\).

#### 2.2 Sensitivity-based rescaling

For structural operational modal analysis a sensitivity-based rescaling method recently has been developed [7]. For an undamped system the sensitivity of the natural frequency \(\omega_n\) of mode \(n\) to a local change in mass in degree of freedom \(i\) is given by [12]
Variable for Structural Modal Analysis | Variable for Acoustic Modal Analysis
--- | ---
$\Phi^s$ | $\Phi^f$
$M^s$ | $M^f$
$K^s$ | $K^f$
$X$ | $P$
$F$ | $Q$
$\bar{m}^s$ | $\bar{m}^f$

Table 1: Analogy between acoustic and mechanical systems

$$\frac{\partial \omega_n^s}{\partial m_i} = -\frac{\omega_n^s}{2 \cdot \bar{m}_n^s} \Phi_{i,n}^{s2}$$  \hspace{1cm} (6)

Solving this equation in a finite difference approximation results in

$$\Delta \omega_n^s = -\frac{\omega_n^s}{2 \cdot \bar{m}_n^s} \cdot \Phi_{i,n}^{s2} \Delta M_i^s$$  \hspace{1cm} (7a)

with

- $\Delta \omega_n^s$ the shift in resonance frequency $\omega_n^s$ of mode $n$
- $\Delta M_i^s$ the change in mass in degree of freedom $i$

If the mass has been changed in $N_c$ degrees of freedom, Equation (7a) becomes:

$$\Delta \omega_n^s = -\frac{\omega_n^s}{2 \cdot \bar{m}_n^s} \left( \sum_{i=1}^{N_c} \Phi_{i,n}^{s2} \cdot \Delta M_i^s \right)$$  \hspace{1cm} (8)

Because the operational mode shapes $\Psi^s$ are not correctly scaled, one can write that each scaled mode shape $\Phi_{i,n}^s$ equals the operational mode shape $\Psi_{i,n}^s$ multiplied by a scaling factor $\alpha_n^s$:

$$\Phi_{i,n}^s = \Psi_{i,n}^s \cdot \alpha_n^s$$  \hspace{1cm} (9)

Finally, the scaling factor can be calculated by combining Equations (8) and (9)

$$\alpha_n^s = \sqrt{\frac{-2 \cdot \bar{m}_n^s \cdot \Delta \omega_n^s}{\omega_n^s \cdot \left( \sum_{i=1}^{N_c} \Psi_{i,n}^{s2} \cdot \Delta M_i^s \right)}}$$  \hspace{1cm} (10)

If one wants to use a mass-normalised scaling-scheme Equation (10) becomes:

$$\alpha_n^s = \sqrt{\frac{-2 \cdot \Delta \omega_n^s}{\omega_n^s \cdot \left( \sum_{i=1}^{N_c} \Psi_{i,n}^{s2} \cdot \Delta M_i^s \right)}}$$  \hspace{1cm} (11)

For Acoustic Operational Modal Analysis the same technique can be used. If one uses the acoustic equivalent variables that are listed in Table 1 Equation (10) becomes:

$$\alpha_n^f = \sqrt{\frac{-2 \cdot \bar{V}_n^f \cdot \Delta \omega_n^f}{\omega_n^f \cdot \left( \sum_{i=1}^{N_c} \Psi_{i,n}^{f2} \cdot \Delta V_i \right)}}$$  \hspace{1cm} (12a)
with
\[
\bar{V}_{n}^{f} = \{\Phi_{n}^{f}\}^{T} \cdot V \cdot \{\Phi_{n}^{f}\}
\]  
(12b)

Now one can use for acoustic mode shapes the volume-normalised scaling-scheme. Equation (12) becomes:
\[
\alpha_{n}^{f} = \sqrt{-2 \cdot \Delta \omega_{n}^{f} \cdot \sum_{i=1}^{N_{c}} \Psi_{i,n}^{f} \cdot \Delta V_{i}}
\]  
(13)

From Equation (13) one can conclude that one now needs to add or subtract some volume instead of some local mass. Doing this and measuring the frequency shift of the resonance frequencies one is able to calculate the scaling factors of the acoustic mode shapes. Note that the second analysis is only needed to estimate the frequency shift and that no mode shape values are needed. So it is not necessary to measure in all the measurement locations! Note also that with this technique it is possible to calculate the FRF’s without using a volumetric acceleration sound source.

3 Simulations and experiments

In this section the proposed procedure will be validated. First it will be validated using a 3-D finite element simulation. In a second step the procedure is validated by an experiment. For both cases the acoustic system was a box with dimensions 1.24 m × 0.3 m × 0.36 m (see Figure 1 for the geometrical configuration).

![Figure 1: box with local volume changes](image)

3.1 3-D computer simulation

In a first simulation the exact solutions have been calculated by means of a finite element analysis (FEA). Then the volume-normalised mode shapes \( \frac{\Phi_{n}^{f}}{\sqrt{V_{n}}} \) were calculated in two ways.

1. The first method used the raw FEA-data and also calculated \( \bar{V}_{n}^{f} \) using the FEA-data and available integration functions.
2. The second method calculated the scaled mode shapes $\Phi^f_n$ with the proposed procedure. So the simulation data of a first simulation (the box without the four volume changes) was compared to the data of the second simulation (the box with the four volume changes) (see Figure 1) using a grid of 90 points (see Figure 2). The frequency shift and the 4 known volume changes were used to calculate the scaling factors $\alpha^f_n$ using Equation (13). Note that the $V^f_n$ too were calculated using the 90 grid points. Using the scaling factors $\alpha^f_n$ and the mode shapes from the first simulation the volume-normalised mode shapes have been calculated.

In Figure 3 the results for the first longitudinal mode (138 Hz) are plotted. Note that the locations of the points are visualised in Figure 2. This plot shows that the sensitivity-based rescaling method results in correctly scaled mode shapes. Figure 4 shows the plot of the mode shape of the second longitudinal mode (277 Hz). This estimate too is quite good.

### 3.2 Experimental validation

In a second step an experimental validation was done. We were mainly interested in the first longitudinal modes. The volume-normalised mode shapes resulting from the rescaling method were compared with those obtained by a FEA. To change the volume locally we used wooden blocks. Each wooden block had a volume of $7.5 \times 10^{-4}$ m$^3$. Three different volume changes were considered: 1 block ($7.5 \times 10^{-4}$ m$^3$), 2 blocks ($1.5 \times 10^{-3}$ m$^3$) and 4 blocks ($3 \times 10^{-3}$ m$^3$). Those three situations are visualised in Figure 5. The results of the FEA are summarised in Table 2.

For the first two modes (138 Hz and 277 Hz) the errors are smaller than 4%. The results of the experiment are summarised in Table 3.

### 4 Conclusions

In this paper it has been shown that it is possible to estimate the acoustic FRF’s using output-only data. The big advantage of this technique is that one does not need to use any
<table>
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<tr>
<th>Volume Change ($m^3$)</th>
<th>Reference Frequency (Hz)</th>
<th>Changed Frequency (Hz)</th>
<th>Frequency difference (Hz)</th>
<th>Estimated Scaled Mode Shape</th>
<th>Error (%)</th>
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Table 2: Results of FEA

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Table 3: Results of experiment
volume acceleration sound sources. Based on a sensitivity analysis it is possible to calculate the scaled mode shapes. Some theoretical aspects have been discussed and the method has been validated by simple 3D-simulations and experiments.

References


(a) pressure visualisation

![Pressure visualization](image)

(b) volume normalised mode shape

![Mode shape graph](image)

Figure 4: Second longitudinal mode shape (277 Hz)


[6] Hermans, L. and Van der Auweraer, H., Modal testing and analysis of structures under operational conditions: Industrial applications, *Mechanical Systems and Signal Pro-
Figure 5: Volume changes


