RATIONAL FRACTION POLYNOMIAL METHOD FOR RANDOM DECREMENT SIGNATURES

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Abstract
The rational fraction polynomial method in combination with the random decrement technique is introduced to provide another choice for the operational modal analysis.

1 Introduction
The rational fraction polynomial (RFP) method [1] is an efficient curve fitting technique based on a rational fraction polynomial form of the frequency response function (FRF). The coefficients in the polynomial form can be determined by means of nonlinear least squares methods to minimize an error function of the measured and fitted FRFs. The fitting procedure used in [1] consists in evaluating the real coefficients of Forsythe orthogonal polynomials when minimizing a frequency-weighted error function. The frequency-weighted error function, however, introduces localization and unbalance effects in the fitted FRF, in spite of giving a linear expression of the error function. To correct such negative effects, Carcaterra and D’Ambrogio [2] introduced the iterative rational fraction polynomial (IRFP) method to minimize the true error function instead of the frequency-weighted error function. The IRFP method expresses the error function as a first order Taylor expansion and allows the use of the same algorithm in the RFP method, including the orthogonal polynomials.

The RD technique was originally proposed by Cole [3] to obtain a single mode RD signature of one single measurement. Following the proposed RD technique, Ibrahim [4] introduced the concept of a multiple-signal RD technique for a multiple degree-of-freedom (MDOF) dynamic system. Similar to the traditional single-signal RD technique, it reduces multiple-mode Gaussian random responses for multiple-measurements to their free vibration responses under certain conditions. These free vibration responses can further be transformed into the frequency domain data in order to identify the modal parameters of the MDOF dynamic system using the IRFP method. However, Huang and Yeh [5] showed that the RD signature of an acceleration response is non-equivalent to the free vibration response, because a singular point exists in the RD signature. The singular point results from the RD signature of the white noise force with respect to the system. The introduction of the IRFP method can reduce the impact of the forced acceleration responses on the identification of the modal parameters of the linear dynamic system.

2 RFP and RD Methods
The definition of the auto- and cross- RD signatures for two ergodic stochastic responses, $X_i(t)$ and $X_j(t)$ is given as follows [6]:

\[
\begin{align*}
R_{i,j}(\tau) &= \mathbb{E}[X_i(t)X_j(t+\tau)] \\
R_{i,i}(\tau) &= \mathbb{E}[X_i(t)X_i(t+\tau)]
\end{align*}
\]
\[ \delta_{X_i}(\tau) = E\left[ X_i(t + \tau) \mid X_i(t) = x_0 \right] \]
\[ \delta_{X_j}(\tau) = E\left[ X_j(t + \tau) \mid X_j(t) = x_0 \right] \]

where \(X_i(t) = x_0\) represents a level-crossing trigger condition.

Use of the rational fraction polynomial (RFP) method is to establish a rational fraction polynomial model that curve-fits the FRF of each RD signature. The FRF \(H_i(j\omega)\) is expressed by the sum of two contributions: \(H_i(j\omega) = H_i(j\omega) + e_i(j\omega)\). \(H_i(j\omega)\) is the true RFP model for the \(i\)th DOF with the ratio of two polynomial coefficients \((a_k, b_k)\),

\[
H_i(j\omega) = \frac{\sum_{k=0}^{p} a_k (j\omega)^k}{\sum_{k=0}^{q} b_k (j\omega)^k}
\]

where polynomial orders, \(p = 2n - 1\) and \(q = 2n\), is set to represent the system of \(n\) vibration modes, and \(b_0 = 1\). \(e_i(j\omega)\) is treated as an error function in the FRF \(H_i(j\omega)\) and this error function is a nonlinear function in terms of the FRF and the coefficients.

### 3 Numerical Simulation and Results

A linear 3DOF dynamic system with the proportional damping matrix was used for the purpose of demonstration. The dynamic system has modes at 0.095 Hz, 0.225 Hz, and 0.378 Hz with 1.5%, 3.5%, 5.9% damping in ascending order. Uncorrelated Gaussian white noise with a zero mean was simulated for three input forces. Each force has a time series of 8000 seconds with a zero mean and a time interval of 0.02 second. The forces were then applied to the dynamic system to give rise to velocity and acceleration responses. The RD signatures were extracted from the responses, when the trigger condition was assigned to each acceleration response.

Use of the IRFP method identified the modal parameters from the FRF matrix based on 3×3 estimated RD signatures for each type of the responses. The identification was conducted in a frequency range from 0 to 0.41 Hz containing the three modes of the dynamic system. The identified modal parameters in comparison with the theoretical modal parameters are shown in Table 1. The relative error, \(\xi = |A_i - \tilde{A}_i|/A_i\times100\%, i = 1...n\), was defined, in which \(A\) and \(\tilde{A}\) are the theoretical parameter and the identified parameter of a mode, respectively. The modal assurance criterion (MAC) was used to quantify the correlation between two mode shapes obtained from the theoretical and identified results [7].

<table>
<thead>
<tr>
<th>Data type</th>
<th>Frequency (Hz)</th>
<th>Error (\xi) (%)</th>
<th>Damping ratio (%)</th>
<th>Error (\xi) (%)</th>
<th>MAC</th>
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<td>11.0</td>
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<td>1.2</td>
<td>4.1</td>
<td>31.5</td>
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</tr>
</tbody>
</table>

### 4 Conclusion

Results show that the modal parameters are well identified from the RD signatures of the acceleration and velocity responses using the proposed method, even though the singular point
exists in the RD signature of the acceleration response. The combination of the RFP and RD methods can give another choice for the operational modal analysis.

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6 References


