Abstract

A half scale physical model (Model Dome) of one of the timbrel domes located in the Nebraska State Capitol is constructed at the Peter Kiewit Institute-Structures Laboratory of the University of Nebraska. The Model Dome represents the unique construction style of concentric timbrel domes. The final dome is to comprise of three-layers of thin clay tiles, however, currently it is formed of a single layer of tiles, as the project is in progress. Using experimental modal analysis techniques, the possibilities of extracting system characteristics are investigated. The system characteristics of interest are natural frequency, mode shapes and component stiffness values. Moreover, several experiments are conducted to study the relationships between these parameters, such as testing the dome with an external mass, i.e. varying the mass, while keeping the stiffness constant. The dome is also tested several times during the year and changes in the material characteristics due to environmental effects are taken into consideration. Three-dimensional finite element models are created for the Model Dome to aid in the material characteristic identification. The comparison of experimental and analytical results also leads to a more accurate simulation of the structural behavior of the actual system and allows for useful recommendations for the finite element modeling of these structures.

1 Introduction

Experimental modal analysis is an effective method to extract several important physical characteristics of a structural system nondestructively, such as the mode shapes (deflected form), the natural frequencies of the modes that are excited, and an approximation for the component and system stiffnesses. These characteristics can also be gathered through a three-dimensional finite element analysis, and the comparison of the experimental and analytical results can lead to a more accurate simulation of the structural behavior of the actual system. This method of model validation for complex masonry structures has been recently developed for unreinforced masonry vaulted systems in Gothic structures by the first author. (Ref [1]) In this study, the developed validation method is extended to timbrel domes. The Nebraska State Capitol in Lincoln, Nebraska, U.S., contains some of the best examples of timbrel vaults and domes designed and built by the Guastavino Company and constructed with the unique Guastavino tiles. A physical half-scale model (Figure 1a- Model Dome) of one of the concentric domes in the Nebraska State Capitol (Figure 1b- Actual Dome) is constructed in order to provide the medium for improving the previously developed method for model updating and system characteristic identification.

While the project carried out by the authors’ team is more extensive and involves numerous experiments and analyses on both the Actual Dome and the Model Dome, this article is limited to discussion of the several experiments that are conducted on the Model Dome only. In this portion of the study, the relationships between the system characteristics such as the mass and stiffness are
evaluated, and an attempt is made to increase the accuracy of the system characteristics
identification. Several tests are carried out such as modal testing with varying mass while keeping
the stiffness constant, or testing at different times during the year to observe changes in the material
characteristics due to environmental effects.

![Figure 1. a) Model Dome constructed at the Peter Kiewit Institute, b) Actual Dome in the Nebraska State Capitol](image)

2 Construction and Brief Description of the Model Dome

The Model Dome is designed through a detailed study of the in situ structure, architectural
drawings, review of the pertinent literature on the timber vault construction techniques (Ref [2];
Ref [3]), and availability of appropriate materials. Similarities and differences between the
currently available building materials and the authentic materials are carefully examined to make
appropriate selections (Table 1).

<table>
<thead>
<tr>
<th>Table 1. Model Dome Components and Materials</th>
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<tbody>
<tr>
<td>Element/Material</td>
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<tr>
<td>------------------</td>
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<tr>
<td>Tiles for the dome webbing</td>
</tr>
<tr>
<td>Side arches and piers</td>
</tr>
<tr>
<td>Mortar Mixture</td>
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</tbody>
</table>

Since authentic Guastavino tiles are no longer manufactured, clay face brick units are used for the
Model Dome. The thickness is the most important dimension of the tiles for the scale model, as it
will determine the thickness of the dome, and have a direct effect on the stiffness and modal
behavior of the system. The clay tiles are 1.25 cm (½-inch) thick and satisfy the dimensions
required for a half-scale model.

The side arches of the Model Dome are constructed from standard clay brick units. While the
thickness of the side arches with a single layer of brick is not half of the thickness of the limestone
arches, this is preferred to a double-ring arch, because multi-ring brickwork arches have inherently different performance characteristics than vousoir arches. (Ref [4]) The adequacy of this smaller thickness for the side arches is checked for stability using a spreadsheet program based on a thrust-line analysis method (Ref [5]) before they are constructed. (Ref [6]) It is also taken into consideration that there are some differences in the resulting dynamic structural behavior between the Model Dome and the Actual Dome in addition to those expected from the change in scale because of the thinner side arch in the scale model. These findings are discussed in another article by the first author, where the dynamic behavior of the Actual Dome and the Model Dome are compared. (Ref [7])

Using the above listed materials, the semi-spherical concentric tile dome is constructed without formwork, which is a unique characteristic of the Guastavino or timbrel dome construction style.

3 Material Properties of the Model Dome

Laboratory testing is performed to gather the individual material properties of the components of the Model Dome. Five masonry prisms for the regular clay brick and Portland cement-lime mortar assembly (Material Set 1), and three prisms for the thin clay tile and Plaster of Paris assembly (Material Set 2) are built. After the specimens cured for twenty-eight days, they are tested for compressive strength ($f'_m$), modulus of elasticity ($E_m$) and Poisson’s ratio ($v$). Testing is performed in accordance with ASTM C1314-03b. (Ref [8]) Using the results of these experiments, arrays of values for each material property are gathered, providing initial values for the finite element models of the Model Dome (Table 2). Poisson’s ratio values are gathered from the ratio of the longitudinal-to-transverse strain data gathered from strain gages applied to each prism. The resulting values are 0.24 and 0.20 for the standard brick and the face brick, respectively. Material density values are determined by buoyancy tests conducted for the clay brick and thin clay tiles. The average density for the clay brick is 2,300 kg/m$^3$ (0.0026 slug/ in$^3$) and for the thin clay tile is 1,800 kg/m$^3$ (0.0020 slug/ in$^3$).

The material properties necessary for a linearly elastic solid model are density ($d$), modulus of elasticity ($E_m$) and Poisson’s ratio ($v$). The sensitivity analyses carried out earlier on similar finite element models show that, among the three material properties, $E_m$ has the largest influence on the dynamic behavior of the system. For this reason, the final validation involves varying the $E_m$ while using the average experimentally achieved density and Poisson’s ratio values. The final modulus of elasticity value is presented in Table 2, while the methodology used to obtain this value is presented later in the article.

<table>
<thead>
<tr>
<th>Material Set</th>
<th>$f'_m$ range from experiments [MPa (psi)]</th>
<th>$E_m$ range based on $E_m = 700f'_m$ [GPa (ksi)]</th>
<th>Modulus of Elasticity</th>
<th>Poisson’s Ratio</th>
<th>Density [Kg/m$^3$ (slug/ in$^3$)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Set 1:</td>
<td></td>
<td></td>
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<tr>
<td>Clay Brick + PCL</td>
<td>9.65-15.86 (1,400-2,300)</td>
<td>6.89-11.03 (1,000-1,600)</td>
<td>10.34 (1,500)</td>
<td>0.24</td>
<td>2,300 (0.0026)</td>
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<td>Mortar</td>
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<tr>
<td>Material Set 2:</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Clay Face Brick +</td>
<td>8.27-11.03 (1,200-1,600)</td>
<td>6.21-7.93 (900-1,150)</td>
<td>6.89 (1,000)</td>
<td>0.20</td>
<td>1,800 (0.0020)</td>
</tr>
<tr>
<td>Plaster of Paris</td>
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<td></td>
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</table>
4 Dynamic Tests on the Model Dome

Several dynamic modes of the Model Dome are observed when small force, low frequency excitations are applied to the structure with an instrumented hammer (PCB Piezotronics ICP® Impulse Hammer Model 086D20). The force sensor in the hammer measures the load input, and the vibrations generated in the structure are measured using seismic shear accelerometers (PCB Piezotronics model 393A03 seismic accelerometer with a sensitivity of 1V/g). Two dynamic signal analyzers are used interchangeably in this study: SIGLAB® and Dactron Photon II®. These two systems have similar characteristics and they generate results that are in agreement; thus the results presented in this paper are not differentiated according to the signal analyzer. After the test data is gathered, the acceleration frequency response function (FRF) is plotted, which is a ratio of the acceleration response to the transform of the driving force. From the FRF functions, several system characteristics are extracted and used in the finite element model updating and validation.

The fixed response data collection method is used in this study. In this setup, a single accelerometer is located at a pre-determined position and the hammer impact is applied at several points on the structure. This method requires two input channels on the signal analyzer (one for the accelerometer and one for the hammer), and it is useful in applications where a single-unit signal analyzer is used or when experiments are conducted at remote locations. Using the modal analysis software STARMODAL® for post-processing, measurements from all of the points are uploaded and combined to display and animate the observed mode shapes on a user created geometric model of the structure (Figure 2), and to list the natural frequency values for the observed modes.

The accuracy of the estimated mode shape increases as the number of data points tested increases. The geometric model of the Model Dome comprises of 65 points, however, as confirmed by experiments, the points around the supports do not move. Therefore, only 48 of the 65 points are actual data points. A single accelerometer is located at the quarter point of one of the quadrants (point 51, Figure 2), and a hammer impact is applied at each of the 48 points. Each test is repeated 3 to 5 times to assure data quality by plotting the coherence (COH) function.

Figure 2. Experimental setups for the Model Dome shown on a 65-point geometric dome model created in STARMODAL

Once all data is gathered and quality is assured, the data files, which comprise of FRFs based on accelerations perpendicular to the surface of the structure, are imported to STARMODAL®, and a list of observed modes (in the form of natural frequencies) and images of the deflected shapes for
each of these modes are gathered. Several experiments are conducted in this study, which are explained in the next sections.

5 Finite Element Model Updating Using Modal Analysis Results

The computer models are created using the finite element analysis software ANSYS. Linearly elastic models are created, since such an assumption is proved to be appropriate within the range of the dynamic loads applied during the experiments. (Ref [1]) Three-dimensional arches, corner piers and the dome-webbing are all modeled as volumes, and meshed with SOLID45 elements. After the models are created and meshed, material properties and boundary conditions are entered. After initial assumptions are made for these characteristics based on laboratory experiments and engineering judgment, respectively, the FEM is updated through the 3-step model validation procedure explained in the next 3 sections.

5.1. Boundary Condition Validation through a Comparison of the Mode Shapes

In a previous study by the first author (Ref [1]), an extensive sensitivity analysis is performed to identify the effect of each parameter entered in the FEM of complex vaulted systems on the resulting modal behavior. A similar study is conducted on the Model Dome and previous findings are confirmed. (Ref [7]) As a result of these analyses, it is seen that the mode shapes are mainly affected by the geometry of the structure and the boundary conditions. Once the correct geometry is established in the model, the first step of the model validation process is to compare the matching experimental and analytical mode shapes, and refine the initial boundary condition assumptions.

The boundary conditions of the structure are very straightforward in this case, as the Model Dome is created in controlled lab conditions, and the entire structure is simulated with the FEM. These boundary conditions are such that the base of each pier (i.e. all 4 support points) is restricted against lateral translation and vertical movement, along with rotations around the three axes. These boundary condition assumptions match well with the actual structure that is constrained at the supports in all degrees of freedom (Figure 1). No other constraints are applied to the model.

The first set of experiments on the Model Dome is conducted soon after the completion of construction and it is used to update the FEM of the undamaged/ initial structure. A comparison of experimental and analytical deformed shapes for the 1st mode is given in Figure 3. Both mode shapes show a bending deformation and maximum vertical displacements at the quarter points of the side arches and the dome. The closely matching resulting mode shapes provide validation of the assumed boundary conditions.

![Figure 3. Comparison of experimental and analytical mode shapes a) Experimental results b) Analytical results](image-url)
5.2. Material Property Validation through a Comparison of the Natural frequencies

Once the corresponding analytical and experimental modes are determined, the natural frequencies for these modes are compared to the analytical values. This part of the procedure provides a validation for the initially entered modulus of elasticity \(E_m\); density \(d\); and the Poisson’s ratio \(v\) values. As mentioned before, based on the sensitivity analyses carried out, among the three material properties, \(E_m\) is found to have the largest influence on the dynamic behavior of the system. For this reason, the averages of the experimentally obtained values for the \(d\) and \(v\) values are used directly, while the \(E_m\) values are varied within the previously established range of values until a reasonable agreement is reached between the analytical and the experimental natural frequency values. When this procedure is applied to the Model Dome, the final material properties listed in Table 2 are achieved. Using these material properties, the experimental values for the first three modes are observed at frequencies of 43.5, 50.6, and 86.7 Hz; while the analytical values for the matching modes are 45.2, 55.6, and 83.4 Hz. The closely matching mode shapes and natural frequencies with 3.9% error for the first mode comparison, and a maximum error of 10% overall, present a good agreement between experimental and analytical results.

5.3. Component and System Stiffness Approximation

The displacement FRF at zero frequency yields the inverse of the stiffness of the structural component that is tested \(k_s\) or the flexibility \(f_s=1/k_s\). This information can also be gathered from the analytical model by finding the displacement per unit force through a static analysis, at a point where the excitation and the response is measured experimentally. The comparison of the values provide the last step of the model updating technique.

In theory, a displacement function can be gathered from the acceleration data, and the component stiffness \(k_s\) can be determined from the reciprocal of the zero frequency point on the displacement FRF function plot. This method is very promising in terms of validating the analytical models for stiffness, however in practice there are problems, since the experimental data is recorded in terms of acceleration and it is necessary to convert it to displacements. Ambient noise results in very small vibration amplitudes at the lower frequencies, and the displacement plot at these lower frequencies becomes unreliable. Nevertheless, if the displacement FRF for a simple system (such as a single degree of freedom system) is inspected, it is observed that the plateau starting at the root of the resonant peak can be extrapolated to 0-Hz and an approximation for the component stiffness is possible. Using a similar simplification, the system stiffness values gathered experimentally and analytically can be compared (Figure 4).

![Model Dome Displacement FRF Plot](image-url)
As can be seen, while the values are in the same order of magnitude, there is a large discrepancy. This is mainly due to the fact that, while the idea is very promising, it is currently a very rough estimation. The authors decided to pursue further work in this area as discussed in the next section.

6 Further Experimental and Analytical Studies on the Model Dome

The authors believe that the relative stiffnesses of the structural components are important characteristics that can be gathered from modal testing and further development of this section of the model validation technique for complex systems has strong merits. Thus, one of the goals of this ongoing study is to improve this portion of the method by investigating alternative ways of measuring stiffness nondestructively. Several experiments are conducted throughout a calendar year, and the results are summarized in this section.

6.1 Results of the second set of experiments after exposure to summer weather conditions

Keeping the testing methodology and the test points constant, a second set of modal experiments is carried out six months after the first set, and during this period the Model Dome was outside, covered by a plastic tarp, enduring a Nebraska summer. Summers in Nebraska are usually dry and extreme temperatures as high as 38-40°C (+100°F) are observed. It is possible that there has been a substantial loss of water in the fast-setting mortar. The measured natural frequency for the first observed mode in this test is 51.5 Hz, an 18% increase over the first set of experimental values. This increase in the natural frequency may be the result of drying plaster of Paris. As the plaster dries, it loses its softness and flexibility and substantial shrinkage cracking occurs when constrained by the clay brick units. During the second set of testing, cracks and lost mortar are observed on the dome, supporting this thesis. Moreover, loss of water, falling of dried mortar, and cracks all mean a decrease in structural mass. The loss of flexibility and reduced mass results in an increased natural frequency, when the basic relationship between natural frequency, stiffness and mass is considered:

\[ \omega = \sqrt{\frac{k}{m}} \]

where, \( \omega \) is the natural circular frequency, \( k \) is the system stiffness, and \( m \) is the system mass.

Another important discussion here is that, while the initially created structure is tested in its linear range using the hammer impacts, the second set of experiments are made on an already cracked structure, even though the cracks are due to environmental conditions and not increased loading. This condition creates a good analogy to the assessments and model updating that are carried out on real structures, where adjustments on material properties must be made based on the age and the actual condition of the structure. Moreover, if the behavior of the cracked structure under external load cases are to be studied after this point (earthquake, wind, etc…), a nonlinear FEM must be created.

6.2 Results of Dynamic experiments with added mass

According to the basic equation given in the previous section, which assumes undamped, forced vibrations in the linear range of the material, an increase in the mass of the structure results in a decreased natural frequency when the stiffness, boundary conditions, and geometry is constant. The mass of the Model Dome is 4630 kg as calculated using the dimensions and densities of the finite element model. An extra mass of 45 kg is applied to the dome in the form of a point load placed at the crown, and the modal experiments are repeated. Starting with the natural frequency of 51.5 Hz measured the same day, the anticipated decrease is very small, around 0.2 Hz, resulting in an estimated natural frequency of 51.3 Hz. The measured frequency during this experiment is 51 Hz, which presents a decrease of 0.5 Hz instead of 0.2 Hz. The error is small at 0.5% and can probably
be eliminated if a larger mass difference between the dome’s total mass and the additional mass is employed. The small change in the total mass (0.09%) is not enough to observe substantial changes or to extract accurate stiffness information from the experiment. The decreasing natural frequency, however, provides more evidence for the suitability of the simplifying assumption of the linear relationship assumed. This preliminary attempt also illustrates the potential of a methodology where the stiffness is kept constant and the total mass is increased substantially to more closely estimate the system stiffness of a complex structure using nondestructive testing methods.

7 Concluding Remarks

Several conclusions are drawn from the studies presented in this article. The model validation technique described has been successfully applied to the physical half scale model (Model Dome) of one of the timbrel domes located in the Nebraska State Capitol. The experimental and the analytical results match closely when previously established material properties are used, pointing out to the accuracy of the geometry and the boundary condition selections of the FEM. Further experiments carried out on the structure suggest that this technique may provide a means to nondestructively assess structural component stiffness. Over time, masonry structures may experience a loss of flexibility and mass due to migration of water from mortars. This loss of flexibility and mass will increase the natural frequency of the structure. Conversely, an increased structural mass should result in a decreased natural frequency for the structure. A preliminary investigation shows that this conjecture seems to hold true, however, the additional mass was small relative to the mass of the structure. The results of these experiments provide evidence to support the assumption of linear structural behavior within the range of dynamic loads applied. Future work will include increasing the additional mass in an attempt to accurately determine the system stiffness.

8 References