DYNAMIC CHARACTERISTICS AND MODELS

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Abstract
Vibration levels of flooring-systems are generally difficult to predict. Nevertheless an estimate may be needed for flooring-systems that are prone to vibrate to actions of humans in motion (e.g. grandstands, footbridges or long-span office floors). One reason for the difficulties is that the dynamic characteristics of a flooring-system do not only depend on material characteristics, floor dimensions and boundary conditions. They are also influenced by the presence of stationary persons on the floor, and these persons may or may not be present. Stationary persons are humans in, for example, sitting or standing posture, and that these persons influence the dynamic characteristics of the floor (floor frequency and floor damping) is demonstrated in the paper. The mechanism of the dynamic interaction between the floor mass and the mass of stationary persons is generally not well understood and the paper therefore looks into this mechanism which is done by carrying out controlled modal identification tests on a test floor. The paper describes the experimental investigations and the basic principles adopted for modal identification. Since there is an interest in being able to model the scenario, attempts are made to calibrate a dynamic model (human-floor system) to experimental findings.

1 Introduction
As indicated in the abstract, the dynamic system considered for this paper is a flooring-system. In the present context, flooring-systems might for example be grandstands or office floors, and such structures may carry stationary crowds of people. If the structures are flexible, they may potentially resonate as a result of actions of humans in motion (for example walking or jumping crowds of people), which may result in unacceptable vibrations in terms of serviceability or safety. Models of the vertical excitation from footsteps and jumping are e.g. proposed in refs. [1] and [2].

As for the dynamic model of a floor carrying stationary humans, it is tempting to model the stationary crowd as a simple added mass (rigid mass model), but in ref. [3] it was suggested that the stationary crowd should be modelled as a mass connected with the structure by springs and dampers. Basically, results in ref. [3] suggested that the crowd of stationary people might act as a SDOF (single-degree-of-freedom) spring-mass-damper system attached to the floor. At least such model was proposed based on field measurements made on a grandstand when empty and when filled with a spectator crowd. Also field measurements reported in ref. [4] seem to indicate that the added mass model for the stationary crowd of people does not work well.

The basic concept of modelling the human whole body as a mass supported by springs and dampers is not new, as it have been adopted in biodynamics (see for example ref. [5]), but it is a relatively novel approach in civil engineering. Hence, a limited number of controlled tests have been made in civil engineering with the purpose of exploring the fundamental mechanisms of the dynamic interaction between stationary humans and the floor mass supporting them. Some exceptions are investigations reported in e.g. refs. [6] and [7]. But on a floor, a crowd of people may take on
different postures. For example, the crowd could be sitting on the floor, or the crowd could be standing. This paper aims at exploring how the two different postures of the crowd influence the dynamic characteristics of a floor. For this purpose a test floor (pin supported hollow-core concrete element weighting almost 6 tonnes) was used, and Figure 1 shows the test floor and the human postures considered in tests.

Besides from experimentally exploring the floor dynamic characteristics as influenced by different human postures, the paper examines the usefulness of two different approaches to modelling the interaction between the floor and crowd mass. These cover a simple added mass model for the crowd (a rigid attachment of crowd mass to the floor mass) and a SDOF spring-mass-damper model for the crowd.

Section 2 presents the two models and the procedures that are used to derive floor dynamic characteristics on the two different crowd model assumptions. Section 3 outlines the test programme including methods used for modal identification of the test floor. Section 4 compares the dynamic characteristics of the floor measured with sitting crowds of people and with standing crowds of people, respectively. Furthermore, the section evaluates the two crowd models in terms of their capability in explaining measured dynamic characteristics of the test floor.

2 Dynamic Models considered and evaluated

Section 2.1 presents the dynamic models of the floor carrying a stationary crowd of people, and Section 2.2 outlines the procedures used when theoretically deriving dynamic characteristics of the systems.

2.1 The Dynamic Models:

Figure 2 gives the two models examined in this paper. The floor is modelled as a grounded DOF carrying a crowd modal mass ($m_2$). In one model, the crowd mass is connected to the floor modal mass ($m_1$) by a spring and a dashpot. In another, the crowd mass is rigidly attached to the floor mass.
Figure 2  Crowd-floor interaction models. Simple added mass model (left) and SDOF spring-mass-damper system for the crowd (right).

The modal mass of the stationary crowd of people \( m_2 \) will be subject to changes in the experimental investigations. The modal mass is the part of the total mass of the crowd that is assumed to participate in vertical vibration in a modal sense. Since, in tests, the crowd will be located at floor midspan, and since the first bending mode of the floor will predominate the vibrations in tests, the crowd modal mass \( m_2 \) will be assumed to equal the weight of the crowd (this parameter is gradually changed in experiments). That only a single mode predominates floor vibrations explains why the floor is modelled as a SDOF system.

The load \( p(t) \) in Figure 2 illustrates the load (an impulsive load), which will cause the floor to vibrate in the experiments with the test floor. For the purpose of the studies of this paper, the mass of the stationary crowd of people is modelled as a single mass for any crowd size (regardless of the number of persons in the crowd) so as to investigate whether this might be reasonable.

Determining the floor dynamic characteristics assuming the simple added mass model is relatively simple, but next it will be explained how dynamic characteristics are derived assuming the 2DOF crowd-floor interaction model.

### 2.2 Extraction of Dynamic Characteristics from the 2DOF Model:

The dynamic characteristics of the combined crowd-floor system (the 2DOF system) can be extracted analytically if all entries in mass, stiffness, and damping matrices of the system are known. In this case, it is possible to solve the quadratic eigenvalue problem using the characteristic equation:

\[
\det(\lambda^2 M + \lambda C + K) = 0
\]

where

\[
M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}; \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}; \quad C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}
\]

Based on this setup, the generally complex roots \( \lambda_j, j = 1, 2 \) can be determined, as described for example in ref. [8]. From the roots \( \lambda_j \), the natural frequencies and damping ratios of the combined (2DOF) system can be calculated using the equations:
\[ f_{(j)} = \frac{\left| \lambda_j \right|}{2\pi}, \quad \zeta_{(j)} = -\frac{\text{Re}(\lambda_j)}{\left| \lambda_j \right|} \]  

(3)

For simplicity, these natural frequencies and damping ratios will be denoted \( f_F \) and \( \zeta_F \) (index \( F \) for floor mode), and \( f_H \) and \( \zeta_H \) (index \( H \) for human mode). The experimental works for this paper provide estimates of \( f_F \) and \( \zeta_F \).

The entries in the system matrices \( \mathbf{M}, \mathbf{C} \) and \( \mathbf{K} \) can be calculated from the modal information that describes the two SDOF subsystems \( (f_i, \zeta_i, m_i; i = 1, 2) \) using eq. (4):

\[ k_i = m_i (2\pi f_i)^2; \quad c_i = 4\pi \zeta_i m_i f_i; \quad i = 1, 2 \]

(4)

where \( f_i, \zeta_i \) and \( m_i \) represent dynamic characteristics of the empty floor and where the parameters \( f_2, \zeta_2 \) and \( m_2 \) represent the dynamic characteristics of the crowd. These six dynamic characteristics of the subsystems are used to construct the eigenvalue problem, which when solved provides the sets \( (f_F, \zeta_F) \) and \( (f_H, \zeta_H) \).

From separate experimental investigations, the dynamic characteristics of the empty floor \( (f_i, \zeta_i, m_i) \) are estimated, and the crowd mass \( m_2 \) is measured for the various crowd configurations employed in tests. The unknowns involved in the calculation of \( f_F \) and \( \zeta_F \) using eqns. (1)-(4) thus reduce to \( f_2 \) and \( \zeta_2 \) being the crowd frequency and crowd damping, respectively.

However, various assumptions (combinations of the set \( (f_2, \zeta_2) \)), can be examined, each assumption resulting in a set \( (f_F, \zeta_F) \) that can be compared with corresponding experimental estimates. This trial-and-error approach is used in the present investigations to explore whether the 2DOF crowd-floor model can explain measured dynamic characteristics of the test floor carrying stationary crowds.

### 3 Test Procedures

#### 3.1 Modal Identification of the Empty Test Floor:

Modal identification tests of the empty floor were carried out prior to embarking on tests with groups of people atop the floor. When groups of people are atop the floor (at midspan) they are expected to be interacting primarily with the first bending mode of the floor, and it is hence reasonable to represent the empty floor as a SDOF system with characteristics corresponding to those of its first bending mode.

Frequency and damping of the first bending mode of the empty floor were derived from free decay tests. An alternative frequency estimate was derived by another approach in which the floor was excited by a small 1 kg concrete block which was dragged across the entire surface of the floor causing the floor to vibrate for 1 hour. In these tests an array of accelerometers picked up the vertical floor response at different positions between floor supports. These test procedures aimed at establishing an excitation random in time and space, as according to [9], these are conditions useful for extracting accurate estimates of dynamic characteristics of the excited system when the system input is not known.

Reassuringly, both modal identification procedures estimated a fundamental frequency of the empty floor in the proximity of 5.8 Hz, and the tests further supported that it is overall reasonable
to determine the modal mass of the floor by assuming a half-sine mode shape function for the first bending mode. The empty test floor is quite lightly damped, as it will appear in Section 4.

### 3.2 Test Sequence, Human-occupied Floor:

A group of people entered the test floor at midspan. While the group was on the floor, the floor was put into free decaying vertical vibrations by an impulse load directed to the floor at midspan. The floor decay was picked up by a displacement sensor which was sampled at a rate of 2400 Hz. A series of decays were recorded in order to enhance the statistical basis for later assessments.

The procedures described above were used for groups of people which were sitting on the floor and for groups which were standing on the floor. For both human postures, decays were recorded for groups of 1, 2, 3, 4, and 5 people, respectively. When sitting on the floor, the persons sat directly on the concrete surface with legs hanging down over the side of the floor. Arms rested in the lap of the person. When standing on the floor, the persons were standing in upright position with arms hanging down freely.

All persons participating in the tests were weighted, which the effect that the weight of each group is known.

### 3.3 Modal Identification of the Test Floor with Occupants:

Frequency and damping characteristics were estimated from free decays using a standard zero crossing algorithm for extraction of frequencies of the oscillations, and the logarithmic decay method provided estimates of damping.

All recorded floor decays were post-processed in this way and this provided variations of floor mode frequency and damping ($f_F$ and $\zeta_F$) with the crowd modal mass ($m_2$). In Section 4, and for simplicity, only mean values of $f_F$ and $\zeta_F$ for each value of $m_2$ are presented.

### 4 Results

This section compares the experimental estimates of floor dynamic characteristics obtained with the standing and with the sitting crowd of people atop the floor. In Section 4.1, the results are aligned with the predictions of the corresponding characteristics estimated assuming the crowd mass to be rigidly attached to the floor mass. In Section 4.2, a spring-mass-damper model for the crowd is considered and evaluated.

#### 4.1 Dynamic Characteristics of the Test Floor:

Floor frequency ($f_F$) and floor damping ($\zeta_F$), and the variation of these parameters with crowd modal mass ($m_2$), were determined in tests with the crowd in standing and in sitting posture on the floor. Figure 3 compares results obtained for the two different postures of the crowd (o: sitting; x: standing). The continuous lines represent the variations predicted assuming a rigid attachment of the crowd mass to the floor mass. The dynamic characteristics of the empty floor are those at $m_2 = 0$. As can be seen, the variations predicted by the simple rigid mass model do not agree well with the experimental results obtained on the test floor. Firstly, a rigid mass does not add damping to the floor (as seen in Figure 3, right), whereas measurements suggest that stationary humans contribute with damping and that floor damping ($\zeta_F$) increases with increases in $m_2$. Secondly, the rigid mass model predicts the well-known floor frequency decline with $m_2$ (seen in Figure 3, left) which only
fairly agrees with the measured floor frequencies \((f_F)\), and not quite well for the standing crowds (+).

![Figure 3](image)

*Figure 3* Variations of floor dynamic characteristics with crowd modal mass. Sitting crowd (○), standing crowd (+) and as predicted by the rigid mass model (continuous lines).

Furthermore, the results suggest that sitting and standing crowds influence the dynamic characteristics of the floor somewhat differently. Particularly, the rates of floor frequency decline with \(m_2\) differ whereas the rates of floor damping increase with \(m_2\) appear to be fairly identical regardless of human posture.

### 4.2 A Spring-mass-damper Model for the Crowds:

As mentioned in Section 2, the floor frequency \((f_F)\) and floor damping \((\zeta_F)\) can be estimated for different values of \(m_2\) assuming a 2DOF crowd-floor interaction model. This requires knowledge of the dynamic characteristics of the empty floor and assumptions to be made related to the frequency and damping of the crowd. Empty floor dynamic characteristics are known, and by trial and error related to the frequency and damping of the crowd, it showed possible to tune 2DOF crowd-floor interaction models such that the results shown in Figure 4 are obtained.

As can be seen, the measured variations, for both the sitting and the standing crowds, can be explained if a SDOF spring-mass-damper system is assumed for the crowd (i.e.
assuming a 2DOF crowd-floor interaction model). At least by such assumption a fair agreement between model
predictions and experimental data can be established (by tuning the parameters of SDOF crowd model) covering both frequency and damping characteristics.

![Graph](image)

**Figure 4** Variations of floor dynamic characteristics with crowd modal mass. Sitting crowd (○), standing crowd (+) and as predicted by tuned 2DOF interaction models (dashed lines: sitting, solid lines: standing).

When tuning the 2DOF system crowd-floor interaction models for the sitting and the standing crowds, respectively, the tuning process was cancelled when a reasonable agreement with experimental data was obtained. A best-fit is therefore not established, but it can be noted that for the crowd of people used in tests for this paper, the crowd appears to behave somewhat stiffer when sitting than when standing. It might thus not be reasonable to model a sitting and a standing crowd with similar dynamic properties.

5 Conclusion

In the experimental investigations for this paper, a group of 5 persons assembled on a test floor which was put into vertical vibration with the crowd atop the floor. In one test, the crowd was standing atop the floor, and in another, the crowd was sitting atop the floor.

The dynamic characteristics of the floor were monitored in both tests, and the results show that a sitting crowd of people and a standing crowd of people influence the floor frequency somewhat differently, whereas a sitting and a standing crowd add almost equal amounts of damping to the floor.
From the results it is obvious that the simple added mass model does not work well in representing the crowd. A SDOF spring-mass-damper model for the crowd showed quite useful in explaining the monitored dynamic characteristics of the floor regardless of whether the crowd was sitting or standing. Generally, the results suggest that the approach of modelling a crowd as a SDOF system attached to a vibrating floor mass seems promising. However, the results also suggest that it may be relevant to let the dynamic characteristics of the SDOF crowd model depend on the posture of the crowd (sitting/standing).

6 Acknowledgements

The author would like to acknowledge M. Vestergaard (graduate student at Aalborg University) for her contribution in making experimental data available for the study presented in this paper. The author would also like to acknowledge students at Aalborg University for participating in tests.

7 References


