Model updating of structures under base excitation and RFM: A comparative study

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ABSTRACT: The Frequency Response Functions (FRFs) sometimes may not be measured accurately due to the difficulties and constraints in the conventional modal testing methods. In these cases, the response functions of structure under base excitation may be available practically. This paper gives a detailed comparison of the results of Model Updating using Base Excitation test data (MUBE) and those of the conventional Response Function Method (RFM) model updating approach. A beam is considered as a test case in a simulated experiment. The convergence of the two methods and the accuracy with which they correct the Finite Element (FE) model are studied. Moreover, the effect of the number of modes on the quality of the updated model is studied. The updated models are compared to obtain the quantity of error in terms of the predicted natural frequencies and response functions.

Keywords: Model Updating, Response Function Method (RFM), Base Excitation, Frequency Response Function (FRF).

1 INTRODUCTION

One of the applications of modal testing is the validation of the FE model of a constructed structure. Once the FE model predicts the dynamic behaviour of structure with an acceptable accuracy, then it can be used for further analysis. However, test results are not usually in perfect agreement with the FE results. Therefore the FE and modal databases need to be reconciled for further analysis. Neither of these two methods can be assumed to be perfect, but if they are combined a more accurate description of the dynamic behaviour of structure can be obtained. Basically, it is believed that the experimental modal model is more reliable than the FE model. Therefore, model updating methods have been developed to improve the FE model using the modal test results.

A number of model updating methods have been proposed in recent years as shown in the surveys by Imregun et al. (1991) and Mottershead et al. (1993). Lin et al. (1995) proposed a method to employ both the analytical and the experimental modal data for evaluating the sensitivity coefficients to improve the convergence of method to the cases where there is a higher error magnitude. There have been attempts to use directly the measured Frequency Response Function (FRF) data for model updating of FE models. A technique named Response Function Method (RFM) has been developed by Lin et al. (1990) uses FRFs to update an FE model. Imregun, Visser and Ewins (1995) and Imregun, Sanliturk and Ewins (1995) devised several methods using simulated and experimental data to show the effectiveness of this technique. Recently, the FRF data based methods have acquired increasing attention of the researchers due to the flexibility these methods offer in the choice of updating parameters.
Lin et al. (2007) developed a new model updating method to update analytical FE models of the structures on which only vibration tests under base excitations can be made to measure response functions. This research work is a detailed comparative study of the RFM and the aforementioned model updating method which uses base excitation results to update a Finite Element model of a beam. The objective of the comparison is to study the convergence of the two methods and the accuracy with which they predict the corrections required in a Finite Element model. The effect of the number of modes on the quality of the updated models is also studied. Moreover, the effects of complete and incomplete experimental data on the updated model are considered. The updated models are compared on the basis of some error indices in terms of the predicted natural frequencies and response functions.

2 BASIC THEORY

A brief description of the Response Function Method (RFM) and Model Updating using vibration test data under Base Excitation (MUBE) are given in this section. Basic formulations of these methods are extended for the fractional correction factors of the physical variables taken as the updating parameters.

2.1 Response Function Method (RFM)

Let \([\alpha_a(\omega)]\) is the receptance of the analytical model and \([\alpha_e(\omega)]\) is that of the experimental model. \([\Delta Z(\omega)]\) is the difference between the dynamic stiffness of the experimental and analytical functions, then we have (Visser 1992):

\[
[\alpha_a(\omega)] [\Delta Z(\omega)] [\alpha_e(\omega)] = [\{\alpha_i(\omega)\}] - [\{\alpha_i(\omega)\}]
\]

Therefore:

\[
[\alpha_a(\omega)] - \omega^2 [\Delta M] + i \omega [\Delta K] + i \omega \{D\} [\alpha_e(\omega)] = [\{\alpha_i(\omega)\}] - [\{\alpha_i(\omega)\}]
\]

After some manipulations, Eq. (2) can be rewritten as:

\[
[C(\omega)] [p] = \{\Delta \alpha(\omega)\}
\]

where \([p] = \{a\}, \{b\}\), in which \(a_i\) is the correction factor corresponds to the \(i\)th element of mass matrix and \(b_i\) is the correction factor corresponds to the \(i\)th element of stiffness matrix. The mass and stiffness error matrices are:

\[
[\Delta M] = \sum_{i=1}^{N} a_i [M^e_i] \quad \text{and} \quad [\Delta K] = \sum_{i=1}^{N} b_i [K^e_i]
\]

where \([M^e_i]\), and \([K^e_i]\), are the \(i\)th element of mass and stiffness matrices, respectively. \(a, b\) are the design parameter changes associated with the \(i\)th element. \(\Sigma\) sign denotes matrix building and not straight summation. \([C(\omega)]\) is a function of the elements of analytical and experimental receptances and the excitation frequency and \([\Delta \alpha(\omega)\] is the difference between the analytical and experimental receptances. The elements of the unknown vector \([p]\) indicate both the amount and the location of the error(s) in the analytical mass and stiffness matrices. By stacking the matrix Eq. (3) for a number (Nf) of different excitation frequencies, the equation becomes over-determined and can be solved for vector \([p]\) using SVD technique.

\[
\begin{bmatrix}
[C_1(\omega)] & \Delta \alpha(\omega_1) \\
[C_2(\omega)] & \Delta \alpha(\omega_2) \\
[C_3(\omega)] & \Delta \alpha(\omega_3) \\
\vdots & \vdots
\end{bmatrix}
= 
\begin{bmatrix}
\Delta \alpha(\omega_1) \\
\Delta \alpha(\omega_2) \\
\Delta \alpha(\omega_3) \\
\vdots
\end{bmatrix}
\]
2.2 Model Updating using vibration test data under Base Excitation (MUBE)

For a structural system with vibrating base, the equations of motion of the system in frequency domain can be written as (Modak et al. 2002):

\[
\left[ [K] - \omega^2 [M] \right] \begin{bmatrix} \{u_0\} \\ \{\hat{u}\} \end{bmatrix} = \begin{bmatrix} \{\hat{f}\} \\ \{0\} \end{bmatrix}
\]

(6)

where \(\{\hat{u}\}\) is the vector of the fixed points and \(\{\hat{u}\}\) is the displacement vector of the unfixed points. \(\{\hat{f}\}\) is the unknown force applied at the base. Suppose that subscripts 1 and 2 correspond to the DOFs of the fixed points and DOFs of the unfixed points respectively, the mass and stiffness matrices can be partitioned as:

\[
[K] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \quad \text{and} \quad [M] = \begin{bmatrix} [M_{11}] & [M_{12}] \\ [M_{21}] & [M_{22}] \end{bmatrix}
\]

(7)

If the error of mass and stiffness matrices can be expressed as linear combinations of the elements of mass and stiffness matrices, we have:

\[
[\Delta M] = \sum_{i=1}^{N} a_i [M^e_i] \quad \text{and} \quad [\Delta K] = \sum_{i=1}^{N} b_i [K^e_i]
\]

(8)

where \([M^e_i]\) and \([K^e_i]\) are the \(i^{th}\) element of mass and stiffness matrices, respectively. \(a_i\) and \(b_i\) are the design parameter changes associated with the \(i^{th}\) element. \(\Sigma\) sign denotes matrix building and not straight summation. From Eq. (8), we have:

\[
\begin{align*}
[\Delta M_{22}] &= \sum_{i=1}^{N} a_i [M^e_{22}] \\
[\Delta M_{21}] &= \sum_{i=1}^{N} a_i [M^e_{21}] \\
[\Delta K_{22}] &= \sum_{i=1}^{N} b_i [K^e_{22}] \\
[\Delta K_{21}] &= \sum_{i=1}^{N} b_i [K^e_{21}]
\end{align*}
\]

(9)

where \([M^e_{22}]\) and \([K^e_{22}]\) are the sub-matrices of \(i^{th}\) element of the mass matrix and \([M^e_{21}]\) and \([K^e_{21}]\) are the sub-matrices of \(i^{th}\) element of the stiffness matrix, which are accordingly expanded and partitioned. Therefore we have (Modak et al. 2002):

\[
\sum_{i=1}^{N} b_i [K^e_{22}] - \omega^2 \sum_{i=1}^{N} a_i [M^e_{22}] [H]_X + \sum_{i=1}^{N} b_i [K^e_{21}] - \omega^2 \sum_{i=1}^{N} a_i [M^e_{21}] [e] = -\left[ [K_{22}]_\lambda - \omega^2 [M_{22}]_\lambda \right] [\Delta H]
\]

(10)

Eq. (10) can be transformed into a set of linear algebraic equations in terms of unknown design parameter changes \(a_i\) (\(i = 1, 2, \ldots, N\)) and \(b_i\) (\(i = 1, 2, \ldots, N\)) as:

\[
\begin{bmatrix} s_1^a & s_2^a & \cdots & s_N^a \\ s_1^b & s_2^b & \cdots & s_N^b \end{bmatrix} \begin{bmatrix} \{a\} \\ \{b\} \end{bmatrix} = -\left[ [K_{22}]_\lambda - \omega^2 [M_{22}]_\lambda \right] [\Delta H]
\]

(11)

where \(s_i^a, s_i^b, \{a\}\) and \(\{b\}\) are:

\[
\begin{align*}
s_i^a &= -\omega^2 [M^e_{22}] [H]_X - \omega^2 [M^e_{21}] [e] \\
s_i^b &= -\omega^2 [K^e_{22}] [H]_X - \omega^2 [K^e_{21}] [e] \\
\{a\} &= \begin{bmatrix} a_1 & a_2 & \cdots & a_N \end{bmatrix}^T \\
\{b\} &= \begin{bmatrix} b_1 & b_2 & \cdots & b_N \end{bmatrix}^T
\end{align*}
\]

(12)
Eq. (11) is established based on the measured response function data under base excitation at one measurement frequency. In a practical vibration test under the base excitation, the response function data are measured at many different measurement frequencies. When the response functions data are used at sufficient number (n) of the measurement frequencies, Eq. (11) can be turned into a set of over-determined algebraic equations, which can be simply written as:

\[
[S][p] = [q]
\]

where

\[
[S] = 
\begin{bmatrix}
  s_1^a(\omega_1) & s_2^a(\omega_1) & \cdots & s_N^a(\omega_1) & s_1^b(\omega_1) & s_2^b(\omega_1) & \cdots & s_N^b(\omega_1) \\
  s_1^a(\omega_2) & s_2^a(\omega_2) & \cdots & s_N^a(\omega_2) & s_1^b(\omega_2) & s_2^b(\omega_2) & \cdots & s_N^b(\omega_2) \\
  \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots \\
  s_1^a(\omega_n) & s_2^a(\omega_n) & \cdots & s_N^a(\omega_n) & s_1^b(\omega_n) & s_2^b(\omega_n) & \cdots & s_N^b(\omega_n)
\end{bmatrix}
\]

\[
{[p]} = \begin{bmatrix} [a] \\
{[b]}
\end{bmatrix} \text{ and } [q] = \begin{bmatrix}
-\left[ [K_{zz}]_A - \omega_j^2 [M_{zz}]_A \right] \{\Delta H(\omega_j)\} \\
-\left[ [K_{zz}]_A - \omega_j^2 [M_{zz}]_A \right] \{\Delta H(\omega_j)\} \\
\vdots \\
-\left[ [K_{zz}]_A - \omega_j^2 [M_{zz}]_A \right] \{\Delta H(\omega_j)\}
\end{bmatrix}
\]

Here \([S]\) is a known coefficient matrix which is formed using the given analytical model and the measured response function data under base excitation. Eq. (13) can be solved for \([p]\) using the linear least square method. The solution \([p]\) is used to reconstruct the updated analytical model together with the original analytical model itself. In the case where multiple nodes are fixed, only the sub-matrices associated with the DOFs of the fixed nodes and unfixed nodes need to be changed through proper partition. Vector \([e]\) which is associated with the DOFs of the fixed nodes will also be changed accordingly.

3 NUMERICAL CASE STUDY

A numerical model of a beam was considered for this study. The dimensions of beam were 1000 mm × 50mm × 5 mm with the modulus of elasticity of 200 Gpa and mass density of 7800 kg/m³ (see Fig. 1). The simulated modal and FRF data, which were treated as experimental data, were obtained by generating a FE model by introducing certain known discrepancies in the thickness of all the finite elements with respect to an analytical model. The details of these discrepancies are given in Table 1. The frequency range of the simulated experiment was 0 to 160 Hz covering four modes. Eqs. (5) and (13) were solved with the move limit of 0.5 for each element of \([p]\). The imposed bounds have no effects on the final cumulative value of the unknown parameters (as calculated from Eq. (14)) during the iterations. The convergence criterion was based on the norm of the difference between two successive vectors of cumulative fractional correction factors. The norm value of 0.05 was used as a convergence criterion for this study.

\[
\bar{p}_j^i = (1 + p_j^i)(1 + p_j^i)(1 + p_j^i) \ldots (1 + p_j^i) - 1
\]

where \(p_j^i\) is \(i^{th}\) element of \([p]\) in \(j^{th}\) iteration and \(p_j^i\) is the cumulative value of \(i^{th}\) element of \([p]\) in \(j^{th}\) iteration.
Two test cases have been considered. In the first case, it was assumed that the measurements were available at all DOFs of the FE model. This case has been referred to as the case of complete data. Thus, one complete column of the FRF matrix and all the eigenvalues and eigenvectors falling in the measurement frequency range are known for the case of complete data. In practice, it is not possible to measure the responses at all DOFs specified in a FE model, either due to the physical inaccessibility or the difficulties faced in the measurement of rotational DOFs. The second case was referred to as the case of incomplete data, where it was assumed that the measurements were not available at all DOFs of the FE model. The incompleteness was considered by assuming that the measurements were available at only translational DOFs. The effect of the quantity of the measured data on the quality of an updated model was studied in this paper. This was done by varying the updating frequency range to cover 1–3 and then 4 modes.

To assess the progress of the iterations, certain model quality indices have been used (Modak et al. 2002). Percentage Average Error in Natural Frequencies (AENF) and percentage Average Error in FRFs (AEFRF) were calculated using the following expressions:

\[
\text{AENF} = \frac{100}{m} \sum_{i=1}^{m} \text{abs} \left( \frac{f_\Lambda - f_X}{f_X} \right)_i \quad (15)
\]

\[
\text{AEFRF} = \frac{100}{nf \times n} \sum_{j=1}^{nf} \sum_{i=1}^{n} \text{abs} \left( \frac{\left[ \text{rec}(f_j) \right]_\Lambda - \left[ \text{rec}(f_j) \right]_X}{\left[ \text{rec}(f_j) \right]_X} \right) \quad (16)
\]

where \(f_\Lambda\) and \(f_X\) are the natural frequencies and \([\alpha]_\Lambda\) and \([\alpha]_X\) are the receptance FRF matrices corresponding to the analytical and experimental model respectively.

4 RESULTS AND DISCUSSIONS

For the case of complete data the RFM and MUBE converged in few iterations and predicted the unknown fractional correction factors to the elements of mass and stiffness matrices. Fig. 2 shows a comparison of the convergence of the cumulative fractional correction factors, when the updating range covers one to four modes only. Due to the completeness of simulated measured data, the convergence was quite rapid and stable, that was, without any excessive or oscillatory variation during the iterations for one to three modes.

For the case of complete data the error indices were calculated before and after updating. Table 2 shows that the error in predicted natural frequencies has been eliminated completely irrespective to the number of modes covered inside the updating range. When only one to four modes were included inside the updating frequency range, the final values of the fractional correction factors were obtained and shown in Fig. 3.
Table 2: Error indices for the case of complete experimental data.

<table>
<thead>
<tr>
<th>No. of modes inside the updating frequency range</th>
<th>AENF</th>
<th>AEFRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before updating</td>
<td>After updating</td>
<td>Before updating</td>
</tr>
<tr>
<td>RFM</td>
<td>MUBE</td>
<td>RFM</td>
</tr>
<tr>
<td>1</td>
<td>5.94</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5.97</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>6.44</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>6.36</td>
<td>0</td>
</tr>
</tbody>
</table>
In the case of incomplete data, the level of error increases due to the reduction in the number of measured coordinates (Table 3). Both methods converge at the same number of iterations except for the case of three modes (Fig. 4). Fig. 5 shows that the final fractional correction factors obtained from both methods when the updating range spans to cover one to three modes. The error levels were not much different for two methods though the MUBE seems to be better for the cases where updating range is limited to include a fewer number of modes. It is observed that the lesser the extent of updating range the more is the error left outside this range. The error levels in predicted natural frequencies have generally decreased as updating range was extended to encompass a greater number of modes.
Table 3: Error indices for the case of incomplete experimental data.

<table>
<thead>
<tr>
<th>No. of modes inside the updating frequency range</th>
<th>Error calculated over the entire measurement range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AENF Before updating</td>
</tr>
<tr>
<td>1</td>
<td>5.94 0.3692 0.0004</td>
</tr>
<tr>
<td>2</td>
<td>5.97 0.0461 0.0003</td>
</tr>
<tr>
<td>3</td>
<td>6.44 0.0035 0</td>
</tr>
</tbody>
</table>

Figure 4: Convergence of the fractional correction factors for the case of incomplete data.

5 CONCLUSIONS

In this paper two methods of model updating namely Response Function Method (RFM) and Model Updating using vibration test data under Base Excitation (MUBE) are compared. The numerical case study on a beam model shows that both methods can predict the error in the FE model with a reasonable accuracy for the cases of complete and incomplete data. For the case of incomplete experimental data the MUBE presents more stability compared to the RFM. Also, MUBE predicts the discrepancies more accurately compared to the RFM especially when the updating range covers a greater number of modes. The convergence can still be obtained in MUBE with the more limited number of modes included inside the updating frequency range compared to the RFM.
Figure 5: Comparison of the final fractional correction factors for the case of incomplete data.

REFERENCES


