Operational modal analysis using ambient Support Excitation: An OMAX Approach

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ABSTRACT: It is shown here that measuring support responses properly in operational modal analysis pays off in terms of accuracy of the identified modal parameters. When using the support responses as inputs for system identification, they are treated as deterministic instead of stochastic quantities, which theoretically reduces the estimation error to zero when no other excitation is present. Since most often, other ambient excitation is important, the recently developed OMAX approach is necessary. The feasibility of using support excitations as deterministic inputs in operational modal analysis is demonstrated with detailed simulation studies.

1 INTRODUCTION

In the context of vibration monitoring, ambient loads are forces that act on a structure in normal operating conditions. While most of these forces can hardly be measured directly, this is not the case for the support excitation: force transducers could be placed at the support interface, or the displacements, velocities, and/or accelerations at the supports could be measured. With the exception of earthquake excitation, this fact has been neglected so far in the operational modal analysis (OMA) literature, chiefly because in most other cases, support excitation is not the only or not the most important ambient load acting on the structure.

However, a recent development in OMA is the possibility to use measured, artificial forces as an addition to the unmeasured, ambient excitation. This combined experimental-operational or OMAX approach (Operational Modal Analysis with eXogenous forces), requires special system identification algorithms, that take both the ambient and the forced excitation into account. The ambient excitation is not considered as noise, as it is in Experimental Modal Analysis (EMA), but as a valuable yet unmeasured part of the excitation, so that the ratio between forced and ambient excitation can be much lower than in classical EMA testing. The modes that are (partly) excited by the measured forces can be scaled in an absolute way, for instance to unity modal mass.

In this paper, the simultaneous use of ambient support responses as measured loads and the other part of the ambient excitation as unmeasured loads, is investigated in an OMAX framework. Section 2 presents the theoretic derivations. In section 3, an extensive simulation study is performed. The considered structure is an industrial process tower that is subjected to two types of ambient loads: wind loading and support excitation due to nearby road traffic. First, it is demonstrated that by considering the response due to the support excitation only, exact identification is possible. Next, the wind loading is added, and the obtained OMAX results are compared with the results from a classical OMA procedure, where all ambient loads are considered as unmeasured.
2 AMBIENT LOADS AND SUPPORT EXCITATION: THEORY

2.1 Support excitation in a linear dynamic finite element model

The Finite Element (FE) method is one of the most common tools for modelling vibrating structures. In case of a linear dynamical model with general viscous damping, one has the following system of ordinary differential equations:

\[ M_i \frac{d^2 v_i'(t)}{dt^2} + C_i \frac{dv_i'(t)}{dt} + K_i v_i'(t) = B_i f_i(t), \]

(2.1)

where \( v_i'(t) \) is the vector with nodal displacements, \( M_i, C_i, \) and \( K_i \) are the mass, viscous damping, and stiffness matrices, respectively, and \( B_i \) is a selection matrix such that the vector with externally applied forces, \( f_i'(t) \), has only elements that are not identically zero. When some of the forces that act on the structure are unknown, but the displacements at the corresponding Degrees Of Freedom (DOFs), called supports, are prescribed, (2.1) can be rearranged to

\[
\begin{bmatrix}
M & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\frac{d^2 v(t)}{dt^2} \\
\frac{d^2 v_2(t)}{dt^2}
\end{bmatrix}
+ \begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}
\begin{bmatrix}
\frac{dv(t)}{dt} \\
\frac{dv_2(t)}{dt}
\end{bmatrix}
+ \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\begin{bmatrix}
v(t) \\
v_2(t)
\end{bmatrix}
= \begin{bmatrix}
B_1 f_1(t) \\
B_2 f_2(t)
\end{bmatrix}
\]

(2.2)

where \( f(t) \) denotes the part of \( f_i'(t) \) containing the known forces, and \( v(t) \) the part of \( v_i'(t) \) containing the unknown displacements. The first block row of (2.2) can be written as

\[ M \frac{d^2 v(t)}{dt^2} + C v(t) = B_3 u(t), \]

(2.3)

where \( B_3 = \begin{bmatrix} B_1 & -M_{12} & -C_{12} & -K_{12} \end{bmatrix} \), and \( u(t) = \begin{bmatrix} \frac{f(t)}{dt} \\
\frac{d^2 v(t)}{dt^2} \\
\frac{dv(t)}{dt} \\
\frac{dv_2(t)}{dt}
\end{bmatrix} \).

If the supports can be considered as fixed, the terms in \( v_2(t) \) and its derivatives drop out from \( B_3 u(t) \). If the supports are moving in a rigid way, the terms in \( v_2(t) \) and \( \frac{dv_2(t)}{dt} \) drop out from \( B_3 u(t) \) when \( v(t) \) is redefined as the displacement relative to the supports [1].

2.2 Conversion to a discrete-time state-space model

By rearranging (2.3) and assuming that \( M \) has full rank, a continuous-time state-space model is obtained:

\[ \frac{dx(t)}{dt} = A_c x(t) + B_c u(t), \]

(2.4)

where \( x(t) = \begin{bmatrix} v(t) \\
\frac{dv(t)}{dt}
\end{bmatrix} \), \( A_c = \begin{bmatrix} 0 & I \\
-M^{-1}K & -M^{-1}C \end{bmatrix} \), and \( B_c = \begin{bmatrix} 0 & M^{-1}B_3 \end{bmatrix} \).

(2.5)

If the output quantities of interest, grouped in a vector \( y(t) \), are linear combinations of nodal
displacements, velocities, or accelerations, one has
\[
y(t) = \left[ C_v - C_v M^{-1} C_v \right] \dot{x}(t) + C_v^T M^{-1} B_s u(t) = C_v x(t) + D_s u(t),
\]
where \( C_v, C_v, \) and \( C_v \) are selection matrices. With a discretization in time with sampling period \( T \), the continuous-time state-space model (2.4,2.6) is converted to a discrete-time model:
\[
x_{k+1} = A x_k + B u_k
\]
\[
y_k = C x_k + D u_k.
\]
\[\text{(2.7)}\]

\[\text{(2.8)}\]

2.3 Incluuing unmeasured loads and sensor noise

The discrete-time state-space model (2.7-2.8) can be enlarged with unobserved loads and output sensor noise:
\[
x_{k+1} = A x_k + B f_k + w_k
\]
\[
y_k = C x_k + D f_k + v_k,
\]
where \( w_k \) and \( v_k \) are noise terms that represent the influence of unmeasured excitations and measurement noise on the outputs, respectively. Since \( w_k \) and \( v_k \) can not be measured, the additional assumption that they have a white noise character, which means that they are uncorrelated over different time instances \( k \), is often made. This assumption is exact when the Power Spectral Density (PSD) of the unmeasured excitations and measurement noise has a flat amplitude over the considered frequency range and a zero phase angle.

2.4 Modal analysis

The undamped eigenfrequencies \( f_{uj} \), damping ratios \( \xi_j \), and mode shapes \( \phi_j \), are obtained from an eigenvalue decomposition of the \( A \) matrix:
\[
A = \Psi \Lambda_d \Psi^{-1}, \quad \phi_j = C \psi_j, \quad \lambda_j = \frac{\ln \lambda_{jj}}{T}, \quad f_{uj} = \frac{\lambda_j}{2\pi}, \quad \text{and} \quad \xi_j = 100 \frac{\Im \left( \lambda_j \right)}{\left| \lambda_j \right|}.
\]
\[\text{(2.11)}\]

where \( \lambda_{jj} \) are the diagonal elements of \( \Lambda_d \) and \( \psi_j \) the corresponding columns of \( \Psi \). From this decomposition, it is clear that when the description (2.7-2.8) or (2.9-2.10) is derived from measurements using system identification techniques, the size of \( A \) equals the number of modes in the considered frequency range, which is much smaller than twice the number of DOFs of a detailed FE model.

3 OMAX USING SUPPORT EXCITATION: SIMULATION STUDY

3.1 Description of the structure and the ambient loading

In order to investigate the feasibility of using measured support responses in an OMAX framework, a detailed simulation study is performed. The considered structure is a six-storey industrial process tower frame (Fig. 1). All vertical and horizontal beams are square reinforced concrete beams \((E = 35 \text{ GPa}, v = 0.2, \rho = 2500 \text{ kg/m}^3)\) with a cross section of \(0.5 \times 0.5 \text{ m}^2\) and a length of \(6 \text{ m}\). The density of the horizontal beams is increased to \( \rho = 3300 \text{ kg/m}^3 \) in order to account for the dead weight of the floors. The diagonals are full steel bars \((E = 210 \text{ GPa}, v = 0.3, \rho = 7850 \text{ kg/m}^3)\) with a diameter of 30 mm when perpendicular to the x-axis, and a diameter of 28 mm when perpendicular to the y-axis. The difference in bar thickness was chosen to break the symmetry, such that double modes and the corresponding undetermined mode shapes are avoided. However, the differences are small, so the structure has pairs of closely spaced
bending modes. All concrete beams are modelled as beam elements with rotational inertia and shear deformation included, and the steel bars are modelled as truss elements. The damping is assumed proportional, where each mode has a damping ratio that is 1% of critical. The structure is clamped at the ground floor. The frequency range of interest is 0-5 Hz. There are six modes in this range (Fig. 1).

<table>
<thead>
<tr>
<th>mode 1</th>
<th>mode 2</th>
<th>mode 3</th>
<th>mode 4</th>
<th>mode 5</th>
<th>mode 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1008Hz</td>
<td>1.1345Hz</td>
<td>1.5970Hz</td>
<td>3.3984Hz</td>
<td>3.5001Hz</td>
<td>4.7842Hz</td>
</tr>
</tbody>
</table>

![Figure 1](image)

Figure 1: First six modes of the tower structure: mode shapes and undamped eigenfrequencies.

The tower structure is instrumented with accelerometers at the third floor and on the roof. Since for the modes of interest, the floors move in a horizontal plane and their deformation is negligible when compared to the overall deformation of the structure (Fig.1), three degrees of freedom per floor are sufficient to characterize the movement of each floor completely. Only two floors are instrumented for demonstration purposes. With this setup, all modes below 5 Hz can be characterized completely (Fig.1).

As a first ambient load, the ground motion at the clamped supports due to freight traffic passing over a traffic plateau on a nearby road is considered (Fig. 2). The trucks are modelled as mass-spring-dashpot systems. Their speed is modelled as a random variable that is uniformly distributed in [25 km/h, 40 km/h]. The road consists of an asphalt top layer (t = 0.15 m, E = 9150 Mpa, ν = 0.33, ρ = 2100 kg/m³), followed by a layer that consists of crushed stone (t = 0.2 m, E = 500 Mpa, ν = 0.5, ρ = 2000 kg/m³) and a bottom layer of crushed concrete (t = 0.25 m, E = 200 Mpa, ν = 0.5, ρ = 1800 kg/m³). It is modelled with 2.5D layered beam elements [2]. The soil layers are modelled as an infinite linear elastic isotropic medium. Two layers are considered: a top layer characterized by a shear wave velocity C_s of 150m/s and a dilatational wave velocity of C_p = 300 m/s, and a semi-infinite halfspace characterized by C_s = 300m/s and C_p = 600 m/s. The shear and dilatational damping ratio of the soil are set to 2%, and the soil density is taken as 1800 kg/m³.

The transfer functions between the force on the traffic plateau and the response at the building supports are calculated using a 2.5D coupled Finite Element-Boudary Element technique [2]. The distance from the tower to the road is 10 m in the y-direction. In the x-direction, the corner of the tower closest to the plateau lies on a 5 m distance from its centre. The forces on the traffic plateau caused by the trucks are calculated by considering the traffic plateau as a fixed rigid body (truck-road interaction is neglected). Multiplying the forces with the transfer functions yields the excitation at the supports of the tower structure. It should be noted that soil-structure interaction is included at the traffic plateau, but neglected at the foundations of the building.

As a second ambient load, wind forces are considered. They are modelled as independent Gaussian random forces with a discrete white character acting on the four corners of each floor of the building in both horizontal directions.
3.2 First simulation study

In a first simulation study, no excitation other than the support excitation due to the truck passages is considered. Five minutes of data have been simulated, where every 30 seconds, a truck with randomized speed passes over the traffic platform. Fig. 3 shows a vertical support excitation due to the first truck passage, as well as a corresponding horizontal acceleration response at one of the floor nodes. Fig. 4 shows the power spectral density of the five minutes excitation signal at one of the supports before decimation with a factor 3. The nonwhite character of the excitation is clear. With the support excitation as inputs and the three measured accelerations at the third and the top floor as outputs, a combined deterministic-stochastic state-space model (2.9-2.10) is identified using the reference-based combined deterministic-stochastic subspace identification (CSI/ref) algorithm [3]. The modal parameters are then calculated using (2.11). Three cases are considered:

- simulation 1a: all displacements, velocities and accelerations at the supports are considered as measured inputs;
- simulation 1b: only the accelerations at the supports are considered as measured inputs, the displacements and velocities are neglected;
- simulations 1c: only the acceleration at one of the supports is considered as a measured input. All other inputs are neglected. This situation boils down to a rigid base excitation assumption.

Figure 3: Vertical displacement at one of the supports due to a truck passage (left) and the corresponding horizontal acceleration response in the x-direction at one of the nodes on the third floor (right).
Table 1: First simulation study: exact undamped eigenfrequencies and relative errors.

<table>
<thead>
<tr>
<th>frequency</th>
<th>1.1007</th>
<th>2.1345</th>
<th>1.2970</th>
<th>3.3984</th>
<th>3.5000</th>
<th>4.7842</th>
</tr>
</thead>
<tbody>
<tr>
<td>error la</td>
<td>6 E -11</td>
<td>4 E -11</td>
<td>1 E -9</td>
<td>1 E -10</td>
<td>3 E -11</td>
<td>1 E -10</td>
</tr>
<tr>
<td>error lb</td>
<td>3 E -5</td>
<td>2 E -5</td>
<td>2 E -5</td>
<td>1 E -6</td>
<td>1 E -5</td>
<td>3 E -5</td>
</tr>
<tr>
<td>error lc</td>
<td>2 E -5</td>
<td>2 E -4</td>
<td>5 E -3</td>
<td>2 E -4</td>
<td>2 E -4</td>
<td>5 E -3</td>
</tr>
</tbody>
</table>

The errors on the identified eigenfrequencies and damping ratios for the three cases can be found in table 1. When all support excitations are correctly taken into account, CSI/ref yields the exact solution. When the displacement and velocity inputs are neglected, the loss of accuracy is insignificant in operational conditions. A rigid base excitation assumption in this case still yields reasonable results, but in many cases it might not, for instance when a bridge is excited at the supports due to traffic underneath. It is therefore clear that measuring all support accelerations pays off in terms of identification accuracy.

3.3 Second simulation study

In a second simulation study, wind loading is added. The wind loads are modelled as uncorrelated Gaussian forces that act in both horizontal directions at all nodes of the floors above ground level. Again three cases are studied for identification with the CSI/ref algorithm [3]:

- simulation 2a: the RMS value of each wind force is set to 0.5 N, which makes that the signal to noise ratio between the response due to support excitation and due to wind excitation in each of the output channels is at around 14 dB;
- simulation 2b: the RMS value is set to 10 N, giving an SNR of around -11 dB;
- simulation 2c: the RMS value is set to 50 N, giving an SNR of around -25 dB.

The errors on the identified eigenfrequencies and damping ratios for the three cases can be found in table 2. It can be seen that in case 2b and case 2c, where the support excitation has only a minor influence on the simulated outputs, the results are similar. This is because the deterministic input signal is very low in both cases, so that the results from the combined deterministic-stochastic system identification approach the stochastic system identification results. However, the results for case 2a, where the support excitation has an important influence on the simulated outputs but the wind excitation is not negligible, are clearly of a higher quality, certainly for the estimated damping ratios. This is due to the fact that the support excitation can be treated as a deterministic input. Stochastic inputs of finite length always cause a variance error on the identified values, even if they obey all other assumptions, which is the case here. On the other hand, deterministic inputs of finite length may yield exact results, as shown in the previous simulation study.
These conclusions are confirmed by the results presented in table 3. They have been obtained by feeding the same data to the SSI-data/ref algorithm [4], which is an output-only system identification algorithm. The results for this OMA approach are clearly much worse than for the OMAX approach for case 2a. For the cases 2b and 2c, there’s no significant difference. For the output only identification, the best results are obtained in cases 2b and 2c, most probably because for these cases the wind excitation, which was modelled as white noise, dominates the support excitation, which doesn’t have a white character (Fig. 4).

| Table 2: Second simulation study: exact undamped eigenfrequencies and relative errors (OMAX) |
|---------------------------------|----------|----------|----------|----------|----------|----------|
| frequency | 1.1007 | 1.1345 | 1.5970 | 3.3984 | 3.5000 | 4.7842 |
| error 2a   | 1 E - 4 | 2 E - 4 | 1 E - 3 | 6 E - 4 | 1 E - 4 | 1 E - 3 |
| error 2b   | 2 E - 3 | 3 E - 4 | 1 E - 3 | 1 E - 5 | 1 E - 3 | 1 E - 3 |
| error 2c   | 2 E - 3 | 6 E - 4 | 1 E - 3 | 8 E - 4 | 2 E - 3 | 1 E - 3 |
| damping ratio | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

| Table 3: Second simulation study: exact undamped eigenfrequencies and relative errors (OMA) |
|---------------------------------|----------|----------|----------|----------|----------|----------|
| frequency | 1.1007 | 1.1345 | 1.5970 | 3.3984 | 3.5000 | 4.7842 |
| error 2a   | 3 E - 3 | 3 E - 3 | 4 E - 2 | 2 E - 3 | 8 E - 3 | 7 E - 3 |
| error 2b   | 2 E - 3 | 2 E - 5 | 5 E - 4 | 1 E - 3 | 1 E - 3 | 1 E - 3 |
| error 2c   | 1 E - 3 | 5 E - 4 | 5 E - 4 | 9 E - 4 | 2 E - 3 | 1 E - 3 |
| damping ratio | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

| Table 4: Second simulation study: exact undamped eigenfrequencies and relative errors (OMA) |
|---------------------------------|----------|----------|----------|----------|----------|----------|
| frequency | 1.1007 | 1.1345 | 1.5970 | 3.3984 | 3.5000 | 4.7842 |
| error 2a   | 7 E - 1 | 9 E - 1 | 3 | 4 E - 1 | 4 E - 1 | 3 E - 1 |
| error 2b   | 5 E - 1 | 4 E - 2 | 3 E - 1 | 2 E - 1 | 1 E - 1 | 2 E - 2 |
| error 2c   | 6 E - 1 | 1 E - 1 | 3 E - 1 | 2 E - 1 | 1 E - 1 | 2 E - 2 |
| damping ratio | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

4 CONCLUSIONS

In this paper, the use of support excitations as measured inputs was investigated in an OMAX framework. From a detailed simulation study, where support excitation due to road traffic was modelled in detail and overall wind excitation was modelled as white noise, it could be concluded that

- when the support excitations have a clear influence on the measured outputs, taking them into account as deterministic inputs in an OMAX framework leads to much more accurate modal parameter estimates than the ones that would have been obtained by output-only identification with the same data. Although theoretically the support displacements, velocities and accelerations need to be taken into account, neglecting the displacements and velocities results in a minor approximation error only;
- when the support excitation is negligible when compared to the other ambient excitation, the OMAX results that are obtained by considering the support excitations as inputs are comparable to the OMA results obtained from the same data.

These observations might result in a more accurate operational modal testing practice in those cases where support excitation is not negligible compared to the other ambient excitation. Bridges and footbridges that overpass highways, and buildings that are close to roads with heavy traffic, are good example cases. Future work will concentrate on the further validation of the presented methodology on operational test data.
REFERENCES


