Identification of operational mode shapes of a soil deposit using a grid of microtremor measurements

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ABSTRACT: This paper presents the evaluation of operational modes shapes, frequencies, and damping of a stiff soil deposit using a grid of microtremor measurements at the surface and the enhanced frequency domain decomposition technique. Eleven mode shapes were clearly identified in an interval of frequencies that ranged from 6.8Hz to 20.2Hz. Damping varied from 0.6% to 0.1% as the frequency increased. The characteristics of the particle motion indicated that these mode shapes represent Rayleigh surface waves travelling at velocities that ranged from 389 m/s to 512 m/s and with wavelengths that varied from 21 m to 60 m. The shear wave velocity and shear stiffness modulus profiles of the soil deposit were obtained from the Rayleigh wave velocities and their wavelengths. These results were validated by seismic up-hole tests. The proposed technique is an efficient and inexpensive option to obtain the dynamic properties of deep soil deposits in comparison to the conventional field tests.

1 INTRODUCTION

One of the most important problems in geotechnical earthquake engineering is to determine the response of a soil deposit to the motion of the rock immediately beneath it. The seismic response depends on the geometry and the mechanical properties of the soil layers and the input motion characteristics. The evaluation of the mechanical soil properties is based on field and/or laboratory tests. Field tests have gained popularity over laboratory tests due to the reduction in sample disturbing and the better replication of the in-situ initial stress conditions.

The most common field tests for high-strain levels are: standard penetration, cone penetration, dilatometer, and pressuremeter. These types of tests are mainly oriented to obtain the strength of the soil layers. On the other hand, low-strain tests are useful to obtain the initial stiffness of the soil layers. These initial stiffnesses determine a very important parameter in ground response analysis: the elastic period of vibration of the soil deposit. The most common low-strain tests are: seismic reflection/refraction, seismic cross/down/up-hole, seismic cone, steady-state vibration, and spectral analysis of surface waves. These tests involve the creation of transient and/or steady-state stress waves at one location and the measurement of the response of the soil deposit at other locations. The main issues associated with field tests are the need of an external dynamic source of waves and the measuring process (large equipment, drilling of boreholes, detection and measuring of wave arrival times, etc).

A very simple and inexpensive way to obtain the fundamental period of vibration of a soil deposit is with microtremor (also known as noise or ambient) measurements and the application of Nakamura’s technique (Nakamura, 1989). This technique has gained popularity because no external dynamic source of waves is required. Other main advantage is that the measurement equipment may consist of only a tri-axial sensor. Nakamura’s technique evaluates the power
spectral ratio between the horizontal (H) and the vertical (V) direction of the recorded microtremor signal. The fundamental period is then obtained from the peak of the H/V plot. The technique has received a lot of criticism due to the uncertainty of the type of waves that produce the peak on the H/V ratio plot. Some researchers claim that the peak is due to Rayleigh surface waves while others affirm that it is due to S body waves (Nakamura 2000). The identification of the type of waves that produce the H/V peak is an important issue since the fundamental period of vibration of interest in ground response analysis is due to S body waves. One of the disadvantages of the Nakamura’s technique is that shear wave velocity profile of the soil deposit can not be obtained from its application. This type of profile is useful for stratigraphic identification of the deposit.

This paper presents the identification of dynamic properties (mode shapes, frequencies and damping) of a stiff soil deposit using a grid of microtremor measurements and the enhanced frequency domain decomposition technique. The testing technique is simple and inexpensive and it can also determine the shear wave velocity and shear stiffness modulus profiles of the soil deposit and the type of waves that are present in microtremor measurements.

2 DESCRIPTION OF THE TEST

2.1 Testing technique

Ambient vibration testing (AVT) is an accurate and cost-effective technique for obtaining modal parameters of large structures such as airplanes, bridges, dams, buildings, etc (Trifunac 1970; Ventura et al.2002). The technique consists of measuring the structure response at different locations to ambient forces such as wind, traffic, human activities, etc. Data is then processed using algorithms of modal operational analysis (OMA), also known as output only modal analysis (Brinker et al.2000; De Moor 2007), to obtain the modal parameters of the structure: natural frequencies, damping, and mode shapes. The main advantages of AVT are: a) equipment for exciting the structure is not needed, b) testing does not interfere with the normal operation of the structure, and c) the measured response is representative of the real operating conditions of the structure (Maalek et al. 2007). The AVT technique will be used here to identify the modal parameters of a geological structure: a soil deposit.

2.2 Measuring equipment

Data quality in AVT depends significantly on the recording system. The equipment generally consists of high sensitive accelerometers (or other types of sensors), cables, and a multi-channel data acquisition system. Some of the inconveniences of the equipment are related to cable handling, sensor balancing, signal clipping, and power supply.

The ambient vibration system of the Earthquake Engineering Research Facility (EERF) at the University of British Columbia (UBC) consists of three wireless Pinocchio WL380 geophone-based units capable of handling long measurements (up to 12hr) at high sampling rates (up to 500sps) without external power supply. Each unit has a GPS antenna for acquiring precise time synchronization and two sets of tri-axial geophones to measure low and high amplitude motion. The geophones have natural frequency of 4.5Hz and 56% damping. The units are programmed using a laptop and secure digital cards. At the completion of the testing, the cards are removed from the units and data is downloaded to a laptop.

2.3 Testing site

The vibration test discussed here was performed on March 2008 at one open field of the University of British Columbia (UBC) in Vancouver, Canada (Fig. 1a). The UBC campus is located in the uplands of the City of Vancouver, considered a zone of high seismic hazard. The site is a heterogeneous glacial deposit (Till) consisting of a compact arrangement of clay, silt, sand and stones ranging from pebble to boulder size. Till is the dominant material in the area with some ailed layers of very dense marine silty sand near the surface. The shear wave velocity for this type of soil deposit varies from 360m/s to 760m/s.

Two types of exploration tests have been done close to the testing area to evaluate the in-situ
mechanical properties. The first one was a Standard Penetration Test (SPT) done 1963. The number of blows per 30cm of sampling (N) varied from N=17 at z=1.0m to N=50 at z=5.0m deep. The test was stopped at 6.0m due to refusal of the hammer when it hit a very hard sandy gravel layer (N>50). The second exploration method was a Seismic Up-Hole test done in 1999. The measured shear wave velocity (Vₛ) varied from Vₛ=400m/s at z=2.0m to 650m/s at z=9.5m deep. At z=10.0m Vₛ dropped to 360m/s and it gradually increased up to 450m/s at z=14.0m deep. This reduction in Vₛ is a characteristic of the heterogeneous nature of glacial deposits. The test was stopped at z=14.0m due to the fact that the drilling process showed the soil conditions were good enough to support the foundation of a structure to be built in the site.

The dimensions of the testing area were 24m x 24m with a 3m x 3m grid, which resulted in 81 testing locations. The WL380 units were programmed to get time histories at each measured location of five-minute duration with sampling rate of 500sp/s. Each unit was placed on a square aluminum plate anchored to the ground by three 10cm-long steel nails (Fig. 1b). One unit was placed in the middle of the area (reference sensor) while the other two (roving sensors) were placed from time to time at the testing locations (Fig. 1c). The total time required for the test was 7 hours.

![Panoramic view](image1.png) ![Pinnochio WL380 unit](image2.png)

![Measurement grid](image3.png)

Figure 1: Testing site and spatial distribution of the measuring locations.

The size grid (ΔL) was chosen so the shortest wavelength (λₛ) of the surface Rayleigh waves was captured at least at five measuring points (ΔL = λₛ/4). Eq. (1) evaluates ΔL according to the
minimum shear wave velocity expected for the testing site ($V_s = 360\text{m/s}$) and the maximum frequency of interest for a seismic response analysis ($F=25\text{Hz}$). Eq. (1) assumes that $V_s$ can be estimated from the velocity of the surface Rayleigh waves as $V_s = 1.1V_R$ (Kramer, 1996).

$$
\lambda_R = \frac{V_R}{F} = \frac{V_s}{1.1 F} \rightarrow \Delta L = \frac{\lambda_R}{4} = \frac{1}{4} \frac{V_s}{1.1} \frac{1}{25\text{Hz}} = \frac{360\text{m/s}}{1.1} = 3.0\text{m} \quad (1)
$$

3 DATA ANALYSIS AND DISCUSSION

The identification of modal vibration properties of the soil deposit was performed by the enhanced frequency domain decomposition (EFDD) technique, implemented in the commercial computer program ARTeMIS®. The technique is based on the approximate decomposition of the system response into a set of independent single degree of freedom (SDOF) systems. Modal parameters are estimated by performing singular value decomposition (SVD) of the power spectral density (PSD) matrix. The identification of SDOF systems is based on the SVD plot and the peak-picking technique. The identified SDOF-PSD functions around the peaks of resonance are taken back to the time domain using the inverse discrete Fourier transform. Then the natural frequencies are obtained by determining the number of zero-crossing as a function of time, and the damping by the logarithmic decrement of the corresponding SDOF normalized auto correlation function (Jacobsen et al, 2007).

Fig. 2 shows the identification of the dominant frequencies of the soil deposit using the SVD plot and the peak-picking technique. The singular value (frequency) at $F \approx 8.5\text{Hz}$, for example, is a peak of resonance and the corresponding eigenvector (mode shape) is plotted in Fig. 3a. The animation of the mode indicates that this is a sinusoidal wave that travels from point 1 to 2. Fig. 3b is the elevation of the mode shape along the direction of propagation. From this figure it was estimated that half of the wavelength of the mode shape is $\lambda/2 \approx 30\text{m}$. The animation of the elevation of the mode shape shows that the direction of particle motion is in the form of a retrograde ellipse. Based on the above, it is concluded that this vibration shape represents a Rayleigh surface wave (Borcherdt 2007) vibrating with a frequency of $F \approx 8.5\text{Hz}$ and with a wavelength of $\lambda_R \approx 60\text{m}$. The velocity of propagation of the Rayleigh waves is $V_R = F \lambda_R$ (Kramer 1996). According to the characteristics found for this mode shape, it is estimated that velocity of propagation of this Rayleigh wave is $V_R \approx 512\text{m/s}$.

![Figure 2: SVD plot of the grid of microtremor measurements.](image)

Fig. 4 shows the modal shape identified at $F \approx 20.2\text{Hz}$. The animation of the 3D view and the elevation indicates that this mode is also a Rayleigh surface wave travelling at $V_R \approx 424\text{m/s}$ and with a wavelength of $\lambda_R \approx 21\text{m}$. Table 1 includes eleven well-defined shapes identified from the
grid of microtremor measurements at UBC campus and the enhanced frequency domain decomposition technique. All the modal shapes correspond to Rayleigh surface waves according to the characteristics of the particle motion. Damping varies from 0.6% to 0.1% as the frequency of vibration increases. The wavelength of the mode at 6.8Hz could not be identified accurately because the dimensions of the testing area were smaller than half of the wavelength. This table also includes the evaluation of the shear wave velocity (Vs) from the Rayleigh wave velocity ($V_R$) according to the equation $V_s = 1.1V_R$ and the evaluation of the shear stiffness modulus (G) from Vs according to the equation $G = \rho V_s^2$ where $\rho \approx 2.2$ Ton/m$^3$ is the density of the Till (Kramer, 1996).

![Diagram](image1)

**Figure 3:** Mode shape identified at $f = 8.5$Hz from the SVD of the grid of microtremor measurements.

![Diagram](image2)

**Figure 4:** Mode shape identified at $f = 20.2$Hz from the grid of microtremor measurements.

<table>
<thead>
<tr>
<th>Order</th>
<th>Mode characteristic</th>
<th>Freq.</th>
<th>$\zeta$</th>
<th>$\lambda_R$</th>
<th>$V_R$</th>
<th>$V_s$</th>
<th>Depth</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rayleigh</td>
<td>6.8</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>2</td>
<td>Rayleigh</td>
<td>8.5</td>
<td>0.6</td>
<td>60</td>
<td>512</td>
<td>558</td>
<td>30</td>
<td>684</td>
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<tr>
<td>3</td>
<td>Rayleigh</td>
<td>10.7</td>
<td>0.3</td>
<td>44</td>
<td>471</td>
<td>513</td>
<td>22</td>
<td>579</td>
</tr>
<tr>
<td>4</td>
<td>Rayleigh</td>
<td>11.8</td>
<td>0.2</td>
<td>42</td>
<td>496</td>
<td>541</td>
<td>21</td>
<td>644</td>
</tr>
<tr>
<td>5</td>
<td>Rayleigh</td>
<td>12.2</td>
<td>0.3</td>
<td>36</td>
<td>439</td>
<td>479</td>
<td>18</td>
<td>504</td>
</tr>
<tr>
<td>6</td>
<td>Rayleigh</td>
<td>13.0</td>
<td>0.2</td>
<td>30</td>
<td>390</td>
<td>425</td>
<td>15</td>
<td>398</td>
</tr>
<tr>
<td>7</td>
<td>Rayleigh</td>
<td>14.5</td>
<td>0.2</td>
<td>33</td>
<td>479</td>
<td>522</td>
<td>17</td>
<td>598</td>
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<tr>
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<td>Rayleigh</td>
<td>16.3</td>
<td>0.2</td>
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<td>408</td>
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<td>13</td>
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<tr>
<td>9</td>
<td>Rayleigh</td>
<td>17.7</td>
<td>0.2</td>
<td>22</td>
<td>389</td>
<td>424</td>
<td>11</td>
<td>396</td>
</tr>
<tr>
<td>10</td>
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<td>19.3</td>
<td>0.1</td>
<td>21</td>
<td>405</td>
<td>442</td>
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<tr>
<td>11</td>
<td>Rayleigh</td>
<td>20.2</td>
<td>0.1</td>
<td>21</td>
<td>424</td>
<td>462</td>
<td>11</td>
<td>470</td>
</tr>
</tbody>
</table>
Fig. 5a shows the wavelengths of the Rayleigh waves ($\lambda_R$) and the shear wave velocities (V_s) for each frequency of vibration shown in Table 1. This type of plot is known as a dispersion curve and it is used in shallow seismic exploration to characterize the variation of the body waves with depth. The estimated V_s correspond to the soil properties at a depth of about $\lambda_R / 3$ to $\lambda_R / 2$ (Kramer, 1996). Fig. 5b plots the shear wave velocity profile obtained from the dispersion curve, assuming that depth $= \lambda_R / 2$. The shallowest depth of exploration with the microtremor measurement technique was 11.0m due to the fact that the shortest wave length detected from the mode shapes was $\lambda_R \approx 21$ m.

Fig. 5b shows that V_s measured with the seismic up-hole test and the Vs evaluated from the microtremor measurements are in good agreement from 11.0m to 14.0m, which was the maximum depth of exploration with the seismic up-hole test. The maximum depth of exploration with the microtremor technique was 30.0m. This figure also shows that V_s evaluated with they microtremor technique increases from 440m/s at 11.0m to 560m/s at 30.0m, which is in a good agreement with the V_s expected for this type of soil deposit. Fig. 5c shows the shear stiffness modulus profile, which is also in a good agreement for the values of G expected for this type of material (Bowles, 1996).

![Dispersion curve](image1)

![Shear wave velocity profile](image2)

![Shear stiffness modulus profile](image3)

Figure 5: Shear wave velocity and Shear stiffness modulus profiles obtained from a grid of microtremor measurements.

Ventura and Thibert (2007) conducted a study to determine the site period of UBC campus according to the Nakamura’s technique and the enhanced frequency domain decomposition.
technique. The three WL380 units were positioned at the same location to get microtremor measurements over a 48 hour period. Both techniques indicated that the site period of UBC campus is $T \approx 0.15$s or $F \approx 6.5$Hz. This frequency is approximately the same frequency obtained with the grid of microtremor measurements in this paper, $F \approx 6.8$Hz. As indicated in Table 1, the motion of the soil deposit at 6.8Hz is due to Rayleigh waves, not to S body waves. This means that the period evaluated with the Nakamura’s technique at UBC is due to Rayleigh waves and it does not represent the site period of UBC for ground response analysis.

The main advantage of getting a grid of measurements is that it is possible to clearly identify the type of waves that cause the motion of the soil deposit. This is done by plotting and animating the mode shape at a given frequency and by identifying the direction of particle motion. Another advantage of the visualization of the mode shapes is the evaluation of the wave length ($\lambda$). This parameter is needed to determine the wave velocity and the depth of influence of the wave into the soil deposit from microtremor measurements. The evaluation of $\lambda$ from a grid of microtremor measurements also opens a new way to study the rotational input motion to structures, a subject that it is usually ignored in earthquake engineering due to the lack of recorded data on the rotational components of strong motion (Teissyer et al, 2006; Lee et al, 2007).

4 SUMMARY AND CONCLUSIONS

This paper presented the evaluation of dynamic properties (mode shape, frequency, and damping) of a stiff soil deposit using a grid of microtremor measurements at the surface and the enhanced frequency domain decomposition technique. Eleven mode shapes were clearly identified in an interval of frequencies from 6.8Hz to 20.2Hz. The characteristics of the particle motion indicated that these modes of vibration represent Rayleigh surface waves travelling at velocities that ranged from 389m/s to 512m/s and with wavelengths that varied from 21m to 60m. The results of the test were validated by comparing the shear wave velocities ($V_s$) obtained from the Rayleigh waves with the $V_s$ measured in the area with seismic up-hole tests. The maximum depth of exploration with the proposed technique was 30.0m.

The Nakamura’s technique indicated that the site period of the stiff soil deposit at UBC is $F \approx 6.5$Hz. This technique assumes that the evaluated site period is due to S body waves. However, the testing technique applied in this paper proved that the motion of the soil deposit at that frequency is due to Rayleigh waves, which means that 6.5Hz is not the site period of the soil deposit for ground response analysis.

The testing technique applied in this paper is simple, inexpensive and accurate to obtain the shear wave velocity profile of deep soil deposits. This information is useful for stratigraphic characterization and ground response analysis.

The information obtained with Standard Penetration and Seismic hole tests is very local and may not represent the whole soil deposit. These tests are also challenging during the drilling stage if the soil deposit has high content of gravel or loose sand or if the exploration is deep. On the other hand, the information obtained with the grid of microtremor measurements is more representative of the whole soil deposit. This technique has also the advantage that the maximum depth of exploration can be easily increased by making the size of the testing area bigger. The type of soil deposit is not a problem for this technique since drilling is not required.

It is recommended to apply the proposed testing technique to other type soil deposits and with different grid sizes and total dimensions of the testing area to detect other types of waves and to study the nature of the waves that produce the peak on the H/V plots of the Nakamura’s technique.

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