Seismic Wave Velocities in Historical Structures: A New Parameter for Identification and Damage Detection

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ABSTRACT: A methodology is introduced to calculate the travel times of seismic waves in structures. The methodology is based on the utilization of envelope functions, calculated via Hilbert transforms. For historical structures, wave travel times provide a better alternative to modal properties for identification and damage detection. An example is presented by using the seismic records from a 1500 year old historical structure in Istanbul.

1 INTRODUCTION

During earthquakes, seismic waves that are generated by the fault rupture travel in all directions, some reaching to the surface of the ground. If the ground surface is free (i.e., no structures), the waves are reflected back into the earth since the air cannot transmit them. When there is a structure, however, the seismic waves continue to propagate into the structure through its foundation, causing the structure to vibrate. The vibrations of structures can, therefore, be studied as a wave propagation problem. Recoded earthquake motions from instrumented structures clearly show this. As an example, Fig. 1 shows a 17-story steel-frame building that are instrumented with four accelerometers at every floor, and its recorded accelerations during a small earthquake nearby (Kohler et al. 2005). It is hard to identify the propagation of seismic waves in the building from the full acceleration records given in Fig. 1. However, if we take a closer look to a one-second long segment (as marked in Fig. 1), the propagation of waves become more visible. This is shown in Fig. 2. The horizontal axes denote the time and the floor level, and the vertical axis is the accelerations deconvolved by the recorded ground accelerations. The accelerations are colour-coded based on their amplitudes. As the figure clearly shows, the incoming seismic waves travel upward in the building reaching to the roof in about 0.4 second. They are then reflected by the free surface on the roof, propagating downward, and again reflected back upwards by the ground. This up and down bouncing of the waves in the building is what causes the vibrations, and they last until the earthquake stops and the vibrations are damped out.

The common approach to study structural vibrations is to utilize modal analysis, where the vibrations are characterized by the natural frequencies, damping ratios, and the mode shapes of the structure. In the wave propagation approach, the vibrations of the structure is characterized in terms of wave velocities, attenuation of wave amplitudes, and the wave reflections and transmissions (Safak 1999). It has been shown that, in terms of system identification, the wave propagation parameters are more reliable and robust when compared to modal parameters, and they are also more sensitive to damage (Safak 1998).

For historical structures, the utilization of wave propagation approach for system identification and damage detection is particularly convenient because in most cases due to the their age, geometry, construction material, and the structural system these structures do not meet the requirements (such as elasticity, linearity, mass and/or stiffness proportional damping) of the clas-
2 INFLUENCE OF DAMP ON WAVE TRAVELS

A critical step in using wave travel times for identification and damage detection is the accurate calculation of wave travel times. This first requires a high-quality and high-sampling-rate recording. The two standard approaches to calculate wave travel times between two recording points have been to use the time differences between characteristic peaks in the signals, or to determine the time lag where the cross-correlation of the signals has a maximum. These methods are acceptable for non-dispersive, non-attenuating media, where the waveforms do not change their shape as they travel. In structures, the waves attenuate due to damping. The attenuation changes the shape (i.e. the phase) of the waves. In other words, the phase shifts in two records are caused by the combined effects of wave travel times, plus the phase distortions due to damping. An impulse given from a lower floor would change its shape as it reaches to the upper floor because of damping. This is schematically shown in Fig. 3. A rigorous theoretical analysis of wave dispersion in an attenuating medium can be found in Aki and Richards (1980).

It is possible to eliminate the phase shifts introduced by damping on the calculated wave travel times. This can be accomplished by using the envelope functions of the signals instead of the signals themselves to calculate the time shifts. The time shifts from the envelope functions can be calculated by observing the time delay between two specific phases, or the peaks of cross-correlation functions. The envelope functions of the signals are calculated by taking their Hilbert transforms and calculating the corresponding analytic functions. The analytic function, \(A[x(t)]\), of a signal, \(x(t)\), is defined by the following equation:

\[
A[x(t)] = x(t) + i \cdot H[x(t)]
\]  
(1)

where \(H[x(t)]\) denotes the Hilbert transform of \(x(t)\), and \(i\) is the unit complex number. \(H[x(t)]\) is defined as

\[
H[x(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(s)}{t-s} \cdot ds
\]  
(2)

The envelope, \(E(t)\), of the signal is defined as the amplitude of the analytic signal, that is:

\[
E(t) = |A[x(t)]|
\]  
(3)

For a narrowband sinusoidal signal, we can show the following:

\[
x(t) = U(t) \cdot \cos(\omega t + \phi)
\]

\[
H[x(t)] = U(t) \cdot \sin(\omega t + \phi)
\]

\[
A[x(t)] = U(t) \cdot e^{i(\omega t + \phi)}
\]

\[
E(t) = U(t)
\]

(4)

It can be shown that for narrow-band signals that are propagating in a frequency-dispersive medium, the phase and group velocities are such that the peaks of the signal and the envelope do not coincide (Bendat and Piersol 1985). Envelope functions are not affected by the dispersive properties of the medium.

This property of envelope functions provide a convenient tool to remove the phase shifts due to damping, and calculate wave travel times more accurately.
3 EXAMPLE

We present an example of calculating wave travel times by using the methodology presented above. The example uses the records from a 1500 year-old historical structure, the Hagia Sophia Museum, in Istanbul, Turkey. Originally constructed as a church between A.D. 532 and 537, Hagia Sophia is famous for its size and massive dome. It was the largest cathedral in the world for nearly a thousand years, until the completion of the Seville Cathedral in 1520.

Historical structures are typically made of stone or masonry blocks joined with mortar and steel ties. Their dynamic behaviour differs significantly from those of modern reinforced concrete structures. In most cases, historical structures are rigid and heavy, do not have much ductility, the dynamic behaviour is mostly nonlinear, and the response includes rigid-body vibration modes influenced by soil-structure interaction. Moreover, under ambient and low-level excitations, the amplitudes of vibrations are small and the signal-to-noise levels in the records are very low. All these make it hard to utilize standard Fourier-based system identification and damage detection techniques that are mostly based on investigating the modal characteristics of the structure. Wave propagation approach provides a better alternative for system identification and damage detection in such structures.

Hagia Sophia Museum is permanently instrumented with 12 acceleration sensors, collecting data continuously in real time (Durukal et al. 2003). Fig. 4 shows the locations of ground sensors and the sensors at the top of the four main pillars of the structure, and the accelerations recorded. These are the records that we will use to calculate the wave travel times in the pillars. The height of the pillars from the ground level to the bottom of the arches are approximately 23 m. The other sensors are located on the arches and not shown in the figure.

To calculate the wave travel times, we first band-pass filter the recorded accelerations around a narrow frequency band centred at the dominant frequency of the structure, and increase the sampling rate of the records to 1000 sps by using interpolation. The original sampling rate in the records (100 sps) is not sufficient for the accurate calculation of wave travel times in 40-m high pillars. Next, we determine the envelope of the filtered accelerations by using the Hilbert transform approach presented above. Filtered ground versus pillar-top accelerations and their envelopes are shown for each pillar in Fig. 5. We then calculate ground to pillar-top wave travel times by using the filtered signals, as well as their envelopes.

The calculated wave travel times are given in Fig. 5. As expected, the wave travel times calculated from the envelope functions are smaller, and represent the actual travel times. The wave travel times calculated from the filtered accelerations are larger, and the difference represents the phase shifts due to damping. A methodology to calculate the damping coefficients that correspond to these phase shifts is given in Safak (1999). The wave travel time for the first pillar is significantly larger than the others because of a known damage in that pillar. The calculated wave travel times correspond to wave velocities of approximately 23 m/s for the first pillar, and 435 m/s for the other pillars.

4 CONCLUSIONS

In historical structures, the wave propagation approach provides a convenient alternative to modal analysis techniques for system identification and damage detection. Because of their geometry, and the material and structural characteristics, most historical structures do not satisfy the assumptions of classical modal behaviour. Wave travel characteristics provide a better insight into the structure. Wave travel times can be calculated accurately by using the envelopes of the band-pass filtered signals, because the envelopes do not contain the phase shifts introduced by damping. A numerical example is presented to introduce the concept by using the records from the historical Hagia Sophia Museum in Istanbul.
REFERENCES


Figure 1: Recorded floor acceleration in a 17-story building during an earthquake.
Figure 2: Propagation of seismic waves during a one-second interval in the 17-story building.

Figure 3: Schematic representation of the effect of damping on the signal’s phase.
Figure 4: Locations of sensors and recorded accelerations in the Hagia Sophia Museum.
Figure 5: Wave travel times calculated from the filtered signals and their envelopes (the difference represents the effect of damping).