A Damage Detection Approach based on Responses Interpolation

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ABSTRACT: This work proposes a new method to detect and locate damage in structures for which recorded responses are available. The main advantage of the proposed method lies in the simplicity of its implementation that does not require any estimation of modal parameters but just responses recorded on the structure in the damaged and in the undamaged (reference) configurations. The method is based on monitoring the changes of the accuracy of a spline function in interpolating the response of the structure between the damaged and the undamaged states. The variation of the interpolation error in a given location is assumed as the damage index in that location. After outlining the proposed method, its feasibility is checked on a numerical example of a beam structure formerly used by other authors considering several damage scenarios. The feasibility of the proposed method is checked for both the case of known input and the case of input unknown. In the two cases different functions can be used to evaluate the interpolation error and results shows that for noise free data the proposed method is able in both cases to detect even very small amount of damage. At the increase of the noise level the sensitivity of the proposed damage index becomes lower but the method is still able to satisfactorily detect damage in the case of known input. If the input is not recorded the method shows a greater sensitivity to noise hence in this case more investigations are needed to make it feasible for practical applications.

1 INTRODUCTION

Thank to the advance in sensor technology, data acquisition systems and data interpretation algorithms, monitoring technique based on the analysis of responses recorded on the structure are becoming a widespread method to monitor structural health conditions (see references 0 for a complete bibliographic report). Measured responses are used to analyze structural behaviour and extract damage sensitive features. In this paper a monitoring technique based on the analysis of time history response of the structure is proposed. The method is based on the availability of: 1) responses recorded in a number of locations of the undamaged structure; 2) responses recorded in the same locations during or after a damaging event. The method does not require any finite element modelling of the structure nor requires knowledge of the mechanical and dynamical characteristics of the structure. Responses are recorded in a limited number of locations and calculated in the other locations via a spline shape function interpolation. The idea underlying the proposed method of damage detection is based on two assumptions. 1) if the spline interpolation is able to predict with a satisfying level of accuracy the response of a structure to a given event, it will be able to predict the response of the same structure to another event with a comparable level of accuracy if the structure is undamaged. 2) If a damage occurs during the second event or is occurred in the time interval between the two events, the accuracy of the in-
interpolation will decrease and it will decrease more for the signals recorded by sensors that are close to the location of damage. In the following, after the description of the proposed method, the proposed method has been herein applied to the numerical model of a continuous beam already used by other authors considering several damage scenarios modelled as stiffness reduction localized in different portion of the structure. Results show the feasibility of the method to detect damage even for small amount of stiffness reduction.

2 THE SPLINE INTERPOLATION AND THE INTERPOLATION ERROR

The response of a beam-like structure in terms of absolute acceleration $\ddot{u}(z,t)$ at location $z$ along the axis, and at time $t$, can be modelled through a spline function that is a function composed by polynomials joined together with continuity conditions (0):

$$s(z,t) = \sum_{j=0}^{d} c_{j}(t)(z-z_{j})^{j}, \quad z \in [z_{j}, z_{j+1}]$$ (1)

The endpoints $z_{j}, z_{j+1}$ of each interval are defined “knots” of the spline. In the use of the spline as interpolating function, the knots are the $n$ locations along the length of the beam where responses are recorded. The unknown coefficients ($c_{0}, c_{1}, c_{2}, c_{3}$) for each interval of the spline function are determined from continuity and interpolation conditions imposed at the internal knots and from boundary conditions at the endpoints (points $z = 0$ and $z = L$).

The interpolation through a spline function gives an estimate $\hat{s}(\xi, t)$ of the real structural response in terms of absolute acceleration $\ddot{u}(\xi, t)$ in a given location $\xi$. For a given number and distribution of recording sensors, such estimate is characterized by an error function of time $e(\xi, t)$ that can be defined as the difference with respect to the real response. Assume that sensors are available in a number of locations along the beam and that a spline interpolation of the responses is calculated considering as knots of the spline all the instrumented locations, except the one where the error of the interpolation is being calculated. This latter knot will be assumed as a “control knots”. The interpolation can be repeated for each location where a sensor is available and assuming this location as a control knot. The difference between the recorded and the interpolated signal in the control knot measures the error of the interpolation in its location, provided that recording sensors are located in all the other locations. To remove the influence of the amplitude and of frequency content of the input on the response, the error function at the control knots can be defined in terms of the magnitude of a frequency response function “recorded” $\tilde{U}(\xi, f)$ (that is calculated through recorded responses) and “interpolated” $\tilde{S}(\xi, f)$ that is calculated through the spline interpolation. In terms of differences in the magnitude of the frequency response function the error at the control knot $\xi$, at frequency $f$, is given by:

$$E(\xi, f) = \tilde{U}(\xi, f) - \tilde{S}(\xi, f).$$ (2)

In order to characterize the error at a given location through a single parameter, the norm $E_{\lambda}(\xi)$ of the function $E(\xi, f)$ over the frequency range of the signal, is calculated:

$$E_{\lambda}(\xi) = \sqrt{\sum_{f=1}^{N} E^{2}(\xi, f)},$$ (3)

being $N$ the length of the sampled signal. The norm $E_{\lambda}(\xi)$ is a measure of the accuracy of the spline interpolation at location $\xi$. The values of $E_{\lambda}(\xi)$ calculated at all the locations $\xi$ of the structure where sensors are available, characterize the current status of the structure.

In terms of the error function $E_{\lambda}(\xi)$, the assumptions on which the method is based can be expressed as follows:

- if the structural condition does not vary, the amplitude of the error function $E_{\lambda}(\xi)$ at the control knots does not change remarkably. On the contrary, a variation of this variable in a certain location $\xi$, reflects a change in the structural condition: the higher the change, the higher the variation of the error $E_{\lambda}(\xi)$.

- the variations of variable $E_{\lambda}(\xi)$ are higher at locations close to the regions where structural changes occur.
If the input to the structure is unknown, the frequency response function cannot be calculated. In this case the transmissibility, instead of the frequency response function, can be used to define the error function in Eq. (2). The transmissibility is the ratio between two responses in the frequency domain between hence in each point of the structure it can be defined as the ratio between the response in that point and the response at the reference point. In the numerical a comparison between the error function calculated basing on the two different functions will be carried out and results discussed with reference to different intensity of noise in the recorded data.

3 DAMAGE DETECTION PROCEDURE

The two assumptions about the error function allow to build a procedure to detect the existence and the location of damage on the base of responses recorded on the structure during two subsequent events inducing vibration of the structure.

Theoretically, if data where not affected by varying operational or environmental conditions, the damage detection feature could be defined as follows.

During vibrations of the structure responses are recorded at selected locations $\tilde{z}$ and the parameters $E_{W_0}(\tilde{z})$ can be calculated in the undamaged configuration. After or during a potentially damaging earthquake, a new evaluation $E_{W_n}(\tilde{z})$ of the error parameters can be performed hence the difference between the values of these parameters during the two events can be obtained:

$$D_E(\tilde{z}) = E_{W_n}(\tilde{z}) - E_{W_0}(\tilde{z})$$

(4)

The absolute value of this difference in a given location $\tilde{z}$ could be assumed as the damage sensitive parameter at location $\tilde{z}$: the higher the changes in the structure, the higher the value of $D_E(\tilde{z})$. If no damage occurs between events 1 and 2 or during event 2, the error function does not change hence $D(\tilde{z}) = 0$; if a variation in the error function occurs, $D(\tilde{z}) > 0$ and this can be an indication of damage.

In the real world, due to several sources of variability influencing recorded responses, the value of the error function $E_{W_n}(\tilde{z})$, hence of the damage index $D(\tilde{z})$ in a given location $\tilde{z}$, can change even if no damage occurs or, vice versa, it can exhibit small changes when damage exist, leading to false or missing detection of damage. In order to take into account this aspect, following the approach proposed in references 0 a statistical classification criteria can be adopted. Each value of the residual error $E_{W_n}(\tilde{z})$ at location $\tilde{z}$ is considered as a realization of a random variable. In the undamaged configuration, under different operational and environmental conditions, this variable is characterized by a certain probability distribution. Damage to the structure causes a significant change of the probability distribution that can be assumed as the damage sensitive feature. In the undamaged configuration, basing on responses recorded ambient vibrations, the mean $\mu_{E_n}(\tilde{z})$ and standard deviation $\sigma_{E_n}(\tilde{z})$ of the distribution of the error function $E_{W_0}(\tilde{z})$ can be calculated at each location $\tilde{z}$. After a potentially damaging event, once the error function $E_{W_n}(\tilde{z})$ has been evaluated at all the locations $\tilde{z}$, a decision should be taken about the possible presence of damage in that location. Note that the variations of function $E_{W_n}(\tilde{z})$ are not able to distinguish between damage occurred at the left or at the right of location $\tilde{z}$. Hence a variation of function $E_{W_n}(\tilde{z})$ indicates a damage in the region adjacent to location $\tilde{z}$. The classification of a certain $\tilde{z}$ as a “damaged location” is carried out through hypothesis testing. The null hypothesis $H_0$ is taken to be “the structure is not damaged at storeys $\tilde{z}$ and $\tilde{z} + 1$”, the alternate hypothesis $H_1$ is “the structure is damaged at storey $\tilde{z}$ or $\tilde{z} + 1$”.

The decision rule needed to assign damage to a given location was chosen to be:

1) choose $H_0$ if $E_{W_n}(\tilde{z}) + \mu_{E_n}(\tilde{z}) < \alpha \cdot \sigma_{E_n}(\tilde{z})$

2) choose $H_1$ if $E_{W_n}(\tilde{z}) + \mu_{E_n}(\tilde{z}) \geq \alpha \cdot \sigma_{E_n}(\tilde{z})$

A direct indication of a state of damage is thus given by positive values of the index:
\[ D(\bar{z}) = \frac{E_k(\bar{z}) + \mu_k(\bar{z})}{\sigma_k(\bar{z})} - \alpha > 0 \]  

(5)

On the basis of the rejection of the hypotheses \( H_0 \) in a statistical sense, this criteria assigns to a given location the tag “damaged” if \( D(\bar{z}) \geq 0 \) and “non damaged” if \( D(\bar{z}) < 0 \). The value of parameter \( \alpha \) defines the threshold between the “damaged state” and “non damaged state” assigned to location \( \bar{z} \) and is chosen basing on the distribution and on the confidence interval required. The choice of the value of \( \alpha \) is always a trade off between the probability of having “false” and “missing” alarms. An increase of \( \alpha \) leads to a reduction in the probability of false alarm while an increase of the threshold is associated to an increase of the probability of missing alarms (see Figure 1).

If \( E(\bar{z}) \) is normally distributed, a threshold equal to 3 times the standard deviation beyond the mean, leads to a confidence level of about 99%; there is only 1% probability that \( E(\bar{z}) \) exceeds the threshold and \( \bar{z} \) is not a damaged location (false alarm).

\[ f_k \]  

UNDAMAGED  

\[ f_k \]  

missing alarm  

DAMAGED

\[ \mu_k + \alpha \sigma_k \]

\[ \mu_k \]

\[ \alpha \sigma_k \]

false alarm

Figure 1: False and missing alarm

Since in this paper the focus was on the general presentation of the method, the construction of the threshold through a rigorous statistical analysis was not performed. The interpolation error was assumed to be normally distributed and a value \( \alpha = 3 \) was considered to calculate the threshold.

4 NUMERICAL APPLICATION

Without loss of generality, the structure described by Choi and Stubbs (see Ref. 0) has been considered to verify the proposed method. Figure 2 reports the numerical model considered for the application. The system properties and the considered damage location and intensity are detailed in reference 0 and are not reported herein due to space limitations.

According to the approach followed in reference 0, responses in terms of absolute acceleration have been assumed to be recorded at the all 61 nodes of the finite element model. Certainly in real world applications such a dense distribution of sensor is not usually deployed. However since in this paper the main aim is to describe the method and to compare results to those obtained in 0, the impact of a more sparse distribution of sensors has not been specifically addressed even if a lower resolution in damage detection should be expected in that case.

Ambient vibrations could be induced by wind, traffic, micro tremors, small earthquakes. In this latter case base excitation can be recorded and the frequency response function in a given location can be calculated as the ratio between the Fourier transforms of the output and of the
input. On the contrary for other sourced of vibrations the excitation is hardly recorded hence only responses are available and the frequency response function cannot be calculated. In order to apply the proposed method in the case of an unknown excitation, frequency response function can be substituted by the transmissibility function defined as the ratio between the Fourier transforms of the responses recorded in two different locations. A reference point can be chosen and the transmissibility function can be calculated in each location with respect to this point. In this case the error function can be defined in each control knot as the difference between the transmissibility function, calculated in the considered point through recorded and interpolated responses.

For the considered example both definitions of the error function have been investigated. In order to simulate response to ambient vibration the structure was subjected to white-noise random excitations of low intensity at its supports. In the application of the proposed method the base input have been considered known in one case and unknown in the other case. In the following the error function calculated through the frequency response function will be denoted as \( E_H(\zeta) \) and the error function calculated through the transmissibility function will be indicated as \( E_T(\zeta) \). In the application \( E_H(\zeta) \) has been calculated assuming as a base excitation the one relevant to the left support while the transmissibility function is calculated assuming as reference point the middle point of the central span.

The first assumption on which the proposed method is based is the independence of the error function from the excitation. In order to check this assumption the values of this function have been calculated in the control knots for 6 different white noise excitations of 400 seconds each. Figure 3 reports the values of function \( E_H(\zeta) \) and \( E_T(\zeta) \) for the structure in the undamaged configuration calculated for the 6 different excitations. In each location the values of the error function remain almost constant in each location for the different excitations. The highest values both of the mean and of the standard deviation of the error function are attained in the region close to the supports of the beam that is in the regions close to the boundary where the spline interpolation loses accuracy. In all the other locations the error function stays pretty constant for all the different excitations. This confirms the assumption of the independence of the accuracy of the spline interpolation from the base input, hence the independence of the error function from the excitation.

![Figure 3: Error function in the undamaged configuration: (a) \( E_H \) (b) \( E_T \)](image)

Using the set of data recovered from the analysis on the undamaged structure the parameters \( \mu_0 \) and \( \sigma_0 \) of the statistical distribution of the error function can be calculated and the threshold between the undamaged and the damages state can be defined as described in section 767 together with the threshold defined to detect possible locations of damage. Of course in a real world application of the method a far higher number of samples would have to be considered to correctly define the parameters of the statistical distribution of the error function considering all the possible different environmental and operational conditions. Basing on the results obtained from this analysis that can only take into account the input variability, the value of the parameter \( \alpha \) used to define the threshold delimiting damaged and undamaged state has been assumed equal to 3 both for \( E_H(\zeta) \) and for \( E_T(\zeta) \).

Damage has been simulated as a reduction of the elastic modulus of one or more elements of the model. The same damage scenarios considered in reference 0 have been considered. The values of function \( D(\zeta) \) calculated according to Eq. (5) are reported in Figure 4-6, for damage to respectively one, two and three elements. In the figures the location and intensity of damage is indicated by the title: D26 4% indicates a 4% reduction of elastic modulus at the element number 26 that spans between elements 26 and 27.
In the figures are compared results obtained through two different evaluations of the error function in Eq. (2). If the input is known, the error function can be calculated as the difference between recorded and interpolated transfer functions \( H \). If, on the contrary the input is unknown, Eq. (2) can be applied with reference to the transmissibility function \( T \) calculated in the frequency domain as the ratio between the response in that location and the response in another location assumed as a reference. For the example considered herein point 31 has been assumed as the reference. However it can be shown that the choice of a different point does not alter the ability of the method to locate damage.

In the figures the damage function calculated by Eq. (5) through the relevant error function \( \varepsilon_{11} \) (or \( E_{11} \)) are reported. Only positive values of function \( D(z) \), that is the ones detecting damage, are reported. The figures show that in all cases except one (see Figure 5c), even for very low percentage of damage, the proposed function is able to correctly locate damage that is the damage function \( D(z) \) is always positive at the locations where damage has been inflicted. In other words for the example considered herein there are no cases of missing alarms except in one case if the transmissibility function is used. In some cases the damage function is positive at non damaged locations (see for example Figure 5a, Figure 6d) leading to a certain number of false alarms. Note however that the amplitude of the damage function (calculated both in terms of the transfer function and in terms of transmissibility function) corresponding to false alarms are always much lower than the ones corresponding to the correct location of damage thus allowing a correct detection of the damaged locations.

As for the ability of the method to give an estimate of the severity of damage, note that due the reduced dimensions of the figures it was not possible to report all the diagrams in the same scale but if this is done, the comparison between cases corresponding to different damage severity shows that the amplitude of function \( D(z) \) increases with the percentage of damage. At this stage of the research no conclusions can be drawn about the relationship between the values of this function and the intensity of damage but the circumstance that the magnitude of this function is somehow proportional to the percentage of elastic modulus reduction makes the proposed function a potentially useful index for damage quantification.

![Figure 4: Damage scenarios with one damaged element](image-url)
Figure 5: Damage scenarios with two damaged elements

Figure 6: Damage scenarios with three damaged elements

An aspect that should be considered when dealing with real data is the effect of measurement noise on the evaluation of the error function. In order to take into account this aspect the time histories recovered from the analysis of the numerical model have been “corrupted” each with a different Gaussian zero mean white noise vector. Two different simulations were carried out considering low and medium noise levels and specifically 1% noise (low noise) and 2% noise (medium noise). The percentages represent the ratio between the root mean square of added noise and the root mean square of the amplitude of the signal.
Due to space limitations the effect of measurements noise on the proposed technique for damage detection will be described with reference to just one of the damage scenarios previously considered. In Figure 7 results relevant to the two considered levels of noise are compared to noise free results (black bars) for the two different definitions of the damage function.

The comparison of the two pictures shows that if the transfer function is used to evaluate the interpolation error (hence $D_H$ is assumed as the damage function), the ability of the method to detect damage is scarcely affected by noise. On the contrary the use of the transmissibility function (hence of $D_T$ as the damage function) is greatly affected by the level of measurement noise and causes a great number of false alarms, making the detection of damage very difficult for a noise level of 1% and almost impossible if the level of noise rises to 2%.

Further investigations are thus needed to reduce the effect of noise on the evaluation of the transmissibility function concentrating the efforts on the correct estimation of the error function hence of the transmissibility function.

5 CONCLUSIONS AND FURTHER DEVELOPMENTS

A method for damage detection basing on responses recorded on the structure during vibrations has been presented. The method is based on the analysis of the variations of an interpolation error of the recorded responses between the undamaged and the damaged configurations. A numerical application carried out using a model previously used by other authors showed the feasibility of the algorithm to both detect and locate even very small amount of damage. If the input to the structure is known and the transfer function is used to define the damage function the method proves robust with respect to measurements noise. In the case the transmissibility function is used to define the damage function more research is needed to reduce the sensitivity to noise. Environmental and operational variability have not been taken into account explicitly in the numerical example presented but a procedure to consider these sources of variability has been proposed and its application to a real case study is a future research effort.

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REFERENCES