A NEW FORMULATION FOR OPTIMUM MAGNITUDE OF ADDITIVE MASS IN SCALING OF MODE SHAPES

S. M. Marashi¹, M. R. Ashory², M. M. Khatibi³

ABSTRACT

One of the most important requirements for design of structures is identification of its dynamic characteristics. For large structures such as bridges, buildings, towers, airplanes, the measurement of ambient forces is difficult or impossible. Therefore, only the response signals can be measured. The methods which identify the modal parameters of structures by using output-only data are called Operational Modal Analysis (OMA) methods. As the structure is excited by unknown forces, the mode shapes cannot be scaled straightforward from the test. A major problem of OMA is that the mode shapes are un-scaled. So far, several approaches have been proposed for scaling the mode shapes. In this paper, the amount of optimum mass change has been obtained for one of the exact scaling formulas using sensitivity analysis of the FEM structure model. It is shown that the error of scaling is minimized by applying this amount of mass change. An FE model of a cantilever beam is used to demonstrate the accuracy of method. A numerical model of beam has been excited by random forces and the responses have been measured in the simulated test. SSI method is applied and the unscaled mode shapes have been obtained. The proposed relation is used to select the optimum mass change for the beam. Finally the mode shapes were scaled and the accuracy of the method has been investigated.

Keywords: Operational modal analysis, Mode shape, Scaling, Mass change

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1. INTRODUCTION

The dynamic analysis of structures is one of the requirements for their design and maintenance. The Finite Element (FE) models of complex structures are not accurate enough due to the errors in the details of geometry, material properties and boundary conditions. Modal testing is known as an experimental alternative for modeling of the dynamic behaviour of complex structures. One drawback of the traditional modal testing is that the excitation of test structures such as bridges and buildings are sometimes impossible or difficult to conduct. Moreover, applying a large force may damage the test structure or cause nonlinear behaviour of the structure [1]. Also the dynamic behaviours of structures such as cars, ships and bridges in-operation differ with their condition during a laboratory vibration test. The environmental noise may also contaminate the force as well as the response signals [2]. In the past few years Operational Modal Analysis (OMA) methods have become a valid alternative for structures where the classic modal testing methods would be difficult to conduct. In OMA which is also known as Ambient Modal Analysis or Output-Only Modal Analysis [2], only the response is measured and the test structure is excited by the ambient forces such as traffic, wind and waves. The first application of OMA were reported for the suspending bridges [3], and vibrating structures without much success [4,5]. Recently more successful procedures have been reported [6-9].

A disadvantage of OMA is that the mode shapes cannot be scaled due to the fact that the ambient excitations cannot be measured. The incompleteness of the modal parameters restricts its applicability in some important applications such as model updating, structural dynamic modification and response prediction. The scaling methods are separated into two categories. In the first group some extra data or some restrictions are required for the method. Doebling et al. used the FE model of structures for scaling the operational mode shapes[10]. Randall et al. considered some limitations in the type of excitation for scaling procedure[11]. Deweer et al. used the forced vibration test on some selected points on the structure in order to scale the operational mode shapes [12]. In the second group the scaling is conducted using only the test data and by changing the stiffness of structure, the mass [13] or both the mass and stiffness [14]. The ambient test is repeated and the scaling factors are estimated by comparing the results [13].

The mass change method was first proposed by Parloo et al. [15] based on the modal sensitivity equations. Brincker et al. proposed an alternative expression derived from the eigen-value equations and the assumption of negligible changes in the mode shapes[16]. Also, additional expressions derived
from the eigen-value equations using both the modified and original mode shapes [13,17]. A different formulation of the mass change method has been proposed by Bernal for the cases where the changes in the mode shapes are not negligible [18]. The projection of the modified mode shape in the original one is used to improve the estimated scaling factors.

A suitable mass change strategy is required in order to scale the mode shapes accurately in which the magnitude, the location and the number of additive masses have to be determined. Such a mass change strategy was proposed by Aenlle et al., who demonstrated that the accuracy of the scaling factors depends on both the accuracy of the identified modal parameter and the mass change strategy[19]. Another mass change approach based on performing several individual mass changes was presented by Fernandez et al. [20]. This approach requires only a small number of masses that are located at different points. The results are then combined to estimate the scaling factors. Further, Aenlle et al. derived an exact set of equations for the scaling factors that by some simplifications[21], all the other formulations such as Bernal can be inferred. So far, no relations are available for optimizing the new formulations such as Bernal. Therefore, in this paper a new method is proposed for selection of additive masses for scaling of operational mode shapes in order to minimize the scaling error of Bernal equation. The method is very useful in achieving the accurate results especially when a large number of modes are considered. The procedure uses sensitivity analysis of the structure numerical model to select the extra masses for minimizing the scaling error. Here, the FE model of structure is applied to perform the sensitivity analysis. A numerical case study is used to demonstrate the effectiveness of the new formula.

2. THEORY

2.1. Scaling of mode shapes using structural modification

For undamped systems, the equation of motion of a structure subjected to a force \( p \) is given by:

\[ m \ddot{u} + k u = p \]  \hspace{1cm} (1)

which results in the eigenvalue equation:

\[ m \phi_0 \omega_0^2 = k \phi_0 \]  \hspace{1cm} (2)

If a dynamic modification, defined by the change in mass and stiffness matrices, is applied to the structure, the new equation of motion becomes:

\[ (m + \Delta m) \ddot{u} + (k + \Delta k) u = p \]  \hspace{1cm} (3)

which provides the following eigenvalue equation for the \( i_{th} \) mode [22-24,13]:

\[ (m \phi_0 + \Delta m \phi_0) A_i \omega_i^2 = (k \phi_0 + \Delta k \phi_0) A_i \]  \hspace{1cm} (4)

The mode shapes of the modified structure \([\phi_i]\) are related to those of the original structure by [21]:

\[ \phi_i = \phi_0 A \]  \hspace{1cm} (5)

From Eq. (5), it follows that the modified mode shapes \( \phi_i \) are expressed as a linear combination of the unmodified mode shapes \( \phi_0 \).

The un-scaled \( \psi_{oi} \) and the scaled or mass normalized \( \phi_{oi} \) mode-shape vectors, corresponding to \( i_{th} \) mode, are related by the expression:

\[ \phi_{oi} = \alpha_{oi} \psi_{oi} \]  \hspace{1cm} (6)

The corresponding equation for the modified structure is given by:
If Eqs. (6) and (7) are substituted in Eq. (5), it results in:

$$\psi_i = \psi_0 \mathbf{B}$$ (8)

which relates the unscaled mode shapes of both systems. From Eqs. (5) and (8) it can be concluded that the matrices $\mathbf{A}$ and $\mathbf{B}$ are related by the equation [21]:

$$\mathbf{B} = \alpha_0 \mathbf{A} \alpha_1^{-1}$$ (9)

If Eq. (4) is pre-multiplied by the un-perturbed mass normalized mode shape vector $\phi_{0j}^T$, it results in:

$$\phi_{0j}^T (m \phi_0 + \Delta m \phi_0) \Lambda_i \omega_{ij}^2 = \phi_{0j}^T (k \phi_0 + \Delta k \phi_0) \Lambda_i$$ (10)

Taking into account the orthogonality properties and the projection of the perturbed mode shapes on the un-perturbed ones given by Eq. (5), Eq. (10) can be expressed as:

$$\left( \omega_{0j}^2 - \omega_{ij}^2 \right) \mathbf{A}_{ji} = \phi_{0j}^T \left( \omega_{ij}^2 \Delta m - \Delta k \right) \phi_{ij}$$ (11)

Finally, if Eq. (9) is substituted in Eq.(11), it becomes:

$$\left( \omega_{0j}^2 - \omega_{ij}^2 \right) \frac{B_{ji} \alpha_{ji}}{\alpha_{0j}} = \alpha_{0j} \alpha_{ij} \mathbf{B}_{0j}^T \left( \Delta k - \omega_{ij}^2 \Delta m \right) \psi_{ij}$$ (12)

From which a closed form expression for the $j$th scaling factor is derived [21]:

$$\alpha_{0j}^2 = \frac{\left( \omega_{0j}^2 - \omega_{ij}^2 \right) B_{ji}}{\psi_{0j}^T \left( \Delta k - \omega_{ij}^2 \Delta m \right) \psi_{ij}}$$ (13)

Eq. (13) is a set of $Nm$ (number of modes) equations which have to be fulfilled for any value of $i$, i.e., there are as many expressions for the scaling factor of mode $j$ as the number of modes considered in the analysis. Another interpretation is that the scaling factor of each mode has to fulfill simultaneously $Nm$ conditions. Moreover, these sets of equations are un-coupled, i.e., only the modal parameters of the $j$th mode are needed in Eq. (13).

### 2.1.1. Bernal equation

If Eq. (13) is particularized for the mass change method ($\Delta k = 0$) and we take $i = j$ it results in:

$$\alpha_{0j}^2 = \frac{\left( \omega_{0j}^2 - \omega_{ij}^2 \right) B_{ji}}{\psi_{0j}^T \Delta m \psi_{ij}}$$ (14)

this equation coincides with that proposed by Bernal [18]. This equation uses the diagonal terms of matrix $\mathbf{B}$ and it will always give very good estimates of the scaling factor even in the cases where the mode shapes change significantly.

As the results of Bernal equation are suitable and no methods have been proposed for minimizing the error of Bernal equation, a new formulation for selection of optimum amount of additive mass is proposed in the following section.
2.2. Optimized scaling of mode shapes

The novelty of this paper is essentially to estimate the magnitude of mass change in order to minimize the scaling error. The method is based on the results of the Finite Element model of structure. The error of scaling as proposed in [25] can be given by:

\[ MSF_i = \frac{\phi_{oi}^T \phi_{oi}}{\phi_{pi}^T \phi_{pi}} \]  

(15)

Here, only the translational DOFs are considered because there are no efficient methods and devices to measure the rotational degrees of freedom [26].

If MSF is equal to one, the scaled mode shapes are completely correlated to the FE mode shapes. Therefore, the error of scaling can be defined by the following equation:

\[ E_i = |1 - MSF_i| \]  

(16)

For N modes of structure, the average error can be given by:

\[ E_{ave} = \frac{1}{N} \sum_{i=1}^{N} E_i \]  

(17)

By substituting Eqs. (6), (15) and (16) in to Eq. (17) the following equation is obtained:

\[ E_{ave} = \frac{1}{N} \sum_{i=1}^{N} \left| 1 - \alpha_{oi}^{-2} \frac{\psi_{oi}^T \psi_{oi}}{\phi_{pi}^T \phi_{pi}} \right| \]  

(18)

By substituting Eq. (14) in Eq. (18), the average error of scaling is given by:

\[ E_{ave} = \frac{1}{N} \sum_{i=1}^{N} \left| 1 - \frac{\left(\omega_{oi}^2 - \omega_{ii}^2\right) B_{ii} \psi_{oi}^T \psi_{oi}}{\omega_{ii}^2 \Delta m \psi_{pi}^T \phi_{pi}} \right| \]  

(19)

If we assume that the equal masses are added to the structure at all its translational degrees of freedom. The mass change matrix reduces to:

\[ \Delta M = \Delta m \ I \]  

(20)

By substituting Eq. (20) into Eq. (19) and considering the relation \( \psi_{oi}^T \Delta m \ I \psi_{ii} = \Delta m \ \psi_{oi}^T \psi_{ii} \). Eq. (19) is simplified to:

\[ E_{ave} = \frac{1}{N} \sum_{i=1}^{N} \left| 1 - \frac{\left(\omega_{oi}^2 - \omega_{ii}^2\right) B_{ii} A_i}{\Delta m \ \omega_{ii}^2 \psi_{oi}^T \psi_{ii}} \right| \]  

(21)

In which:

\[ A_i = \frac{\psi_{oi}^T \psi_{oi}}{\phi_{pi}^T \phi_{pi}} \]  

(22)

For minimizing Eq. (21), we have:

\[ \frac{\partial E_{ave}}{\partial \Delta m} = 0 \]  

(23)
If the structure is linear, the total change of mode shape is equal to change of mode shape due to each additive mass. By using Eq.\((20)\) the modified mode shape vector is given by:

\[
\psi_{II} \equiv \psi_{0i} + \sum^{L}_{k=1} \frac{\partial \psi_{0i}}{\partial m_k} \Delta m_k \equiv \psi_{0i} + \Delta m \sum^{L}_{k=1} \frac{\partial \psi_{0i}}{\partial m_k}
\]  

(25)

The derivation of Eq.\((25)\) to \(\Delta m\) can be given as:

\[
\frac{\partial \psi_{II}}{\partial \Delta m} \equiv \sum^{L}_{k=1} \frac{\partial \psi_{0i}}{\partial m_k}
\]  

(26)

Moreover, the sensitivity of the \(i\)th unmodified mode shape to the mass change at the \(k\)th DOF, as derived by [15] can be given by:

\[
\frac{\partial \psi_{0i}}{\partial m_k} \equiv -\frac{\phi_{ki}^{2}}{2} \psi_{0i} + \psi_{kt} \sum^{N}_{t=1,t \neq i} \frac{\omega_{ti}^{2}}{\omega_{iti}^{2} - \omega_{ti}^{2}} \Phi_{kt} \Phi_{t}
\]  

(27)

If the structure is linear, similarly, the total change of natural frequency is equal to the change of natural frequency due to each additive mass. By using Eq.\((20)\) the modified natural frequency vector is given by:

\[
\omega_{II} \equiv \omega_{0i} + \sum^{L}_{k=1} \frac{\partial \omega_{0i}}{\partial m_k} \Delta m_k \equiv \omega_{0i} + \Delta m \sum^{L}_{k=1} \frac{\partial \omega_{0i}}{\partial m_k}
\]  

(28)

The derivation of Eq.\((28)\) to \(\Delta m\) can be given as:

\[
\frac{\partial \omega_{II}}{\partial \Delta m} \equiv \sum^{L}_{k=1} \frac{\partial \omega_{0i}}{\partial m_k}
\]  

(29)

Moreover, the sensitivity of the \(i\)th unmodified natural frequency to the mass change at the \(k\)th DOF, as derived by Parloo et al. [15] can be given by:

\[
\frac{\partial \omega_{0i}}{\partial m_k} \equiv -\omega_{0i} \frac{\phi_{ki}^{2}}{2}
\]  

(30)

Using Eqs.\((8)\),\((28)\),\((29)\) and some mathematical calculations the following equations can be derived as:

\[
\omega_{0i}^{2} \equiv \left( \omega_{0i} + \Delta m \sum^{L}_{k=1} - \omega_{0i} \frac{\phi_{ki}^{2}}{2} \right)^{2} \equiv \omega_{0i}^{2} \left( 1 + \Delta m \sum^{L}_{k=1} - \frac{\phi_{ki}^{2}}{2} \right)^{2}
\]  

(31)

\[
\frac{\partial \omega_{0i}^{2}}{\partial \Delta m} \equiv 2 \omega_{0i}^{2} \left( 1 + \Delta m \sum^{L}_{k=1} - \frac{\phi_{ki}^{2}}{2} \right) \sum^{L}_{k=1} \frac{\phi_{ki}^{2}}{2} - \frac{\phi_{ki}^{2}}{2}
\]  

(32)
\[
\frac{\partial B_{ii}}{\partial m} = Q_i F_i \tag{33}
\]

in which:

\[
Q_i = \text{the } i_{th} \text{ row of } [\psi_0]^{-1} \tag{34}
\]

\[
F_i = \sum_{k=1}^{L} \frac{\partial \psi_{0i}}{\partial m_k} \tag{35}
\]

Combining Eqs. (25-33) and Eq. (24) yields:

\[
\begin{align*}
\frac{1}{N} \sum_{i=1}^{N} \left[ 1 - \frac{\left(1-\left(1+\Delta m c_i\right)^{2}\right)Q_i \left(V_i + \Delta m F_i\right) A_i}{\Delta m \left(1+\Delta m c_i\right)^{2}Z_i \left(V_i + \Delta m F_i\right)} \right] & \left( \frac{2 c_i \left(Q_i \left(V_i + \Delta m F_i\right) \right) A_i}{\Delta m \left(1+\Delta m c_i\right) \left(Z_i \left(V_i + \Delta m F_i\right)\right)} \right) \\
- \frac{\left(1-\left(1+\Delta m c_i\right)^{2}\right)Q_i \left(V_i + \Delta m F_i\right) A_i}{\Delta m \left(1+\Delta m c_i\right)^{2}Z_i \left(V_i + \Delta m F_i\right)} & = 0 \tag{36}
\end{align*}
\]

in which:

\[
C_i = \sum_{k=1}^{L} \frac{\phi_{ki}^2}{2}, \quad \psi_{0i} = V_i, \quad \psi_{0i}^T = Z_i \tag{37}
\]

Eq. (35) gives the optimum mass change for scaling of the mode shapes obtained from OMA methods.

2.3. Stochastic Subspace Identification method

The Stochastic Subspace Identification (SSI) method uses a state space model of structure, in which 2nd order problem is converted to the 1st order problem as [27]:

\[
x_{t+1} = Ax_t + w_t \tag{38}
\]

\[
y_t = Cx_t + v_t \tag{39}
\]

\[y_t\] is the output that is generated by the process noise, \(w_t\) and the measurement noise \(v_t\). The dynamics of the physical system is modeled by the state matrix \(A\). The observable part of system dynamics is extracted from the state vector by forward multiplication of the observation matrix \(C\) [28]. \(x_t\) is the Kalman sequence that is found in the SSI method by an orthogonal projection technique.

\[
x_{t+1} = Ax_t + K_t e_t \tag{40}
\]

\[
y_t = Cx_t + e_t \tag{41}
\]

By performing an eigenvalue decomposition of the matrix \(A\) as \(A = V \mu V^{-1}\) and introducing a new state vector \(z_t = V^{-1} x_t\), Eq. (40) and Eq. (41) can also be written as Eq. (42) and Eq. (43):

\[
z_{t+1} = \mu z_t + \Psi e_t \tag{42}
\]

\[
y_t = \Phi z_t + e_t \tag{43}
\]
where \( \mu \) is a diagonal matrix holding the discrete poles related to the continuous time poles \( \lambda_i \) by \( \mu_i = \exp(\lambda \Delta t) \), and where the matrix \( \Phi \) is holding the left-hand mode shapes and the matrix \( \Psi \) is holding the right-hand mode shapes [27]. The final results are achieved by a singular value decomposition of the full observation matrix and extracting a subspace holding the modes in the model. This will lead to Unweighted Principle Component (UPC), which is one of the SSI estimation classes [29]. Subsequently, the modal parameters of structure can be achieved using stabilization diagram which is a tool that shows poles of system versus different orders [27].

3. **NUMERICAL CASE STUDY**

3.1. Finite Element model of a cantilever beam

The numerical model of a cantilever beam was built using the Finite Element Method (FEM) based on the relations in [30] (Fig. 1). Each node had two degrees of freedom (DOFs). The specifications of beam are given in Table 1.

![Figure 1 Model of free-clamped beam](image)

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Thickness (mm)</th>
<th>Density (kg/m(^3))</th>
<th>Young's modulus (GPa)</th>
<th>No. of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>49.3</td>
<td>6.48</td>
<td>7850</td>
<td>200</td>
<td>20</td>
</tr>
</tbody>
</table>

First fifteen natural frequencies and mode shapes of beam were estimated using the FEM given in Table 2.

3.2. Modal parameter identification using SSI method

The simulated operational modal testing was conducted by exciting the beam using random excitation. The responses are measured and SSI method is applied on measured responses. The stabilization diagram is plotted shown in Figure 2.

![Figure 2 Stabilization diagram of beam](image)

The natural frequencies (Table 2) and mode shapes were estimated using SSI method.
Table 2 Comparison of the natural frequencies (Hz) of beam from FEM and SSI

<table>
<thead>
<tr>
<th>Mode#</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td>5.28</td>
<td>33.11</td>
<td>92.72</td>
<td>181.70</td>
<td>300.39</td>
<td>448.82</td>
<td>627.08</td>
<td>835.33</td>
</tr>
<tr>
<td>SSI</td>
<td>5.28</td>
<td>33.12</td>
<td>92.80</td>
<td>181.79</td>
<td>300.26</td>
<td>448.76</td>
<td>626.95</td>
<td>835.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode#</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td>1073.78</td>
<td>1342.78</td>
<td>1642.77</td>
<td>1974.33</td>
<td>2338.19</td>
<td>2735.18</td>
<td>3166.23</td>
</tr>
<tr>
<td>SSI</td>
<td>1074.32</td>
<td>1342.79</td>
<td>1643.22</td>
<td>1973.49</td>
<td>2337.44</td>
<td>2735.46</td>
<td>3166.80</td>
</tr>
</tbody>
</table>

The comparison of the mode shapes from SSI and FEM based on the MAC criterion is shown in Fig.3.

Figure 3 MAC criteria between SSI and FEM mode shapes

Although the mode shapes from SSI and FEM are completely correlated based on the MAC criterion (Fig. 3), but the mode shapes from SSI, are not scaled as shown in Fig. 4.

Figure 4 Comparison of the mode shapes of the beam

As a result, a scaling factor has to be introduced for each mode shape derived from OMA. This can be concluded by inspecting the MAC criterion in Fig. 3.

3.3. Scaling the mode shapes using mass change method

The unscaled mode shapes from SSI would be scaled using the mass change method. The extra masses were added to the beam, shown in Fig. 5.

Figure 5 Beam with added masses
To scale the mode shapes of beam the amount of mass change was selected using the proposed method in section 2-2. To assess the feasibility of method, the scaling procedure was conducted using different mass loadings (Table 3) which are selected arbitrarily.

<table>
<thead>
<tr>
<th>#</th>
<th>1</th>
<th>2</th>
<th>3 (Optimum amount)</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δm(g)</td>
<td>20</td>
<td>40</td>
<td>62</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

The extra masses (Table 3) were added to the beam and the mode shapes were scaled using Eq.(14) and the Modal Scaling Factor (MSF) Error between the scaled and unscaled mode shapes were calculated using Eq.(15). Table 4 indicates the average error of scaling for different amount of mass change and the optimum mass change.

<table>
<thead>
<tr>
<th>#</th>
<th>1</th>
<th>2</th>
<th>3 (Optimum amount)</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Error</td>
<td>13.2</td>
<td>1.57</td>
<td>0.94</td>
<td>1.60</td>
<td>1.31</td>
</tr>
<tr>
<td>Maximum Error</td>
<td>67.21</td>
<td>12.33</td>
<td>4.08</td>
<td>8.57</td>
<td>5.26</td>
</tr>
</tbody>
</table>

The errors for the optimum mass change versus each mode are given in Fig. 6. The results show that the proposed formula can minimize the average error of scaling.

![Figure 6](image)

**Figure 6** Error of scaling for optimum mass change

### 4. DISCUSSION

The equations such as Bernal are more accurate than the others as $B_{ij}$ has the mode shape alteration inherently. When different amount of mass changes added to the structure and if a few numbers of modes are engaged in the calculation, almost identical acceptable results would be obtained using Bernal equation. When large number of modes is considered, the error increases significantly due to truncation error of parameter $B_{ij}$. In this work an approach is presented for determining the optimum amount of mass change for the Bernal equation. As was seen in section 3.3 accurate results were obtained by applying the optimum mass change method even if a large number of modes were considered.

### 5. CONCLUSION

In this paper, a new approach for selection of the amount of mass change in scaling of OMA mode shapes has been introduced. A priori knowledge of the mode shapes from Finite Element analysis is required for selecting the optimum added mass. The method was numerically applied to a cantilever beam. The beam was excited using random signals and the responses were measured in a simulated test. SSI method was applied on the measured responses and the unscaled mode shapes were obtained.
The proposed scaling procedure was applied and scaling factors were estimated. The results show that the average error of scaled mode shapes is minimized when the optimum mass change is used.

REFERENCES


