OPERATIONAL MODAL ANALYSIS FROM FLIGHT TEST DATA

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ABSTRACT
In the framework of the operational modal analysis (OMA), several approaches have been developed for estimating the modal parameters, i.e., natural frequencies, damping ratios, and mode shapes. In this paper, two methods for the estimate of the modal parameters from ambient responses are proposed. The first one estimates the modal parameters directly from the correlation functions, the other from the power spectral densities. The main purpose is to increase the reliability of the results and to speed up the estimation procedure: two fundamental tasks in dealing with flight tests. In the first case the Hilbert transform allows the reconstruction of the analytical signal of the correlation functions and then the exponential least square fitting can be implemented for the evaluation of the modal parameters. On the other side, a similar procedure recovers the causality of the power spectral densities by applying the Hilbert transform and then the least square polynomial ratio technique gives the modal estimates. The validation of these methodologies against standard input-output tests is performed on simple cantilever beam. Then, the capabilities of the proposed approaches to identify the dynamic properties of a flying system, represented by a Savannah ultra-light aircraft, are reported. For this purpose, the evolution of the poles of the dynamic system to the change of the flight conditions are investigated.

Keywords: Flight Test, Correlation and Power Spectral Density functions, Hilbert Transform

1. INTRODUCTION
The dynamic characterization of an aircraft plays a leading role in the design process and needs that both numerical simulations and experimental investigations are carried out simultaneously in order to correctly predict the behavior during the operative conditions, [1]. The system modal parameters (natural frequencies, damping ratios and mode shapes) representing the signature of the structure, can be used to characterize resonance problems and to track the evolution of the dynamic properties of state-dependent mechanical systems, as required for the flutter instability clearance of flying aircraft, [2, 3]. In this latter case the stability of the coupled structure-inertial-aerodynamic system can be investigated by analyzing the evolution of the real part of the poles, strictly related to the natural frequencies and damping ratios, [4]. The advantages in using the operational modal analysis lie in the

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capability of estimating the modal parameters in the effective working condition of the aircraft, yielding high accuracy and good reliability in the flight tests, [5]. The current state-of-the-art shares the common basic idea that the modal parameters can be achieved by manipulating the correlation functions or the related Fourier transform functions (the power spectral densities) if the excitation can be assumed as white noise, [6, 7, 8]. In the effort to widen the field of applicability of the OMA techniques to flight tests for aeroelastic identification, two approaches, based either in the time- or in the frequency-domain, are presented in this paper. The main hypothesis on which both methods rely on is that the dynamic random excitation, acting on a linear system, is considered “almost” of stochastic type with a constant spectra (white noise assumption), at least in the frequency band of interest, as all the operational modal analysis methods. Because of the finite length observation time, correlation and power spectral densities functions of the output responses could not be considered as causal functions, [9]. Considering as an example, the auto-correlation functions are generally real even functions as the corresponding auto-power spectral density functions. Because commercial codes use models with complex and conjugate poles for functions with real and imaginary parts when fitting the data, some difficulties could arise when fitting data with polynomial ratios. Therefore, a direct estimate of the modal parameters from such functions could lead to wrong estimates. In this paper the problem of the causality is solved by introducing the Hilbert transform in the estimating process. As a result, the modal parameters could be efficiently and accurately estimated using the causal correlation functions or the causal power spectral density functions of the output responses from an exponential least square fitting in time domain or a least square polynomial ratio technique in the frequency domain, commonly available in numerical routines. The accuracy of the estimates, with respect to standard input-output methods, have been evaluated both numerically and experimentally. In this paper, only some results from the experimental activities are reported for conciseness. The proposed approaches are applied to track the evolution of the dynamics of the Savannah ultra-light aircraft from the output response accelerations, recorded on board the aircraft at different flight conditions, in terms of the dependency of the system poles from the flight speed and the height.

2. THEORETICAL BACKGROUND

The proposed approaches are mainly based on the capability to reconstruct a causal correlation function matrix or power spectral density (PSD) matrix from a limited time recorded signal. In the following subsections some details on the theoretical basis are described for both the time and frequency domain approaches.

2.1. Time domain approach, OTD

In this approach, called Operational modal analysis method in Time Domain (OTD), the response measured at the i-th degree of freedom of the structure, \( y_i(t) \), is used to evaluate the auto-correlation function, \( \Phi_{i,i}(\tau) \). Following [9], because such a function is an (real) even function, it is possible to write:

\[
\Phi_{i,i}^e(\tau) = 2\Phi_{i,i}(\tau)
\]

where the superscript e is for the even part of the considered function. The odd part of such correlation function could be evaluated by introducing the signum function, \( sgn(\tau) \). Calling with the superscript o the odd part, then:

\[
\Phi_{i,i}^o(\tau) = \Phi_{i,i}^e(\tau)sgn(\tau)
\]

Therefore the complete analytical auto-correlation function, \( R_{i,i}(\tau) \), is given by:

\[
R_{i,i}(\tau) = \frac{\Phi_{i,i}^e(\tau)+\Phi_{i,i}^o(\tau)}{2}
\]
When the measurements refer to different locations, then the causal cross-correlation function, $\Phi_{y'y'}(\tau)$ could be evaluated from the well known relationship:

$$R_{y'y'}(\tau) = \Phi_{y'y'}^e(\tau) + \Phi_{y'y'}^o(\tau) \quad (4)$$

in which the even and the odd part of the cross-correlation functions are gained from:

$$\Phi_{y'y'}^e(\tau) = \frac{\Phi_{y'y'}(\tau) + \Phi_{y'y'}(-\tau)}{2}$$

$$\Phi_{y'y'}^o(\tau) = \frac{\Phi_{y'y'}(\tau) - \Phi_{y'y'}(-\tau)}{2}$$

In such equations the quadrature part of the cross-correlation functions is obtained, in turn, by Hilbert transforming the measured functions, that is $\Phi_{y'y'}(-\tau) = \mathcal{H}\left[\Phi_{y'y'}(\tau)\right]$, [10, 11]. Once the causal correlation matrix is derived, then the modal parameters could be estimated, using a Least-Squares Complex Exponential based fitting algorithm, applied to the following governing equation:

$$R_{yy}(\tau) = \int_0^\infty \int_0^\infty U(t_1)\Lambda(t_2)U^{-1}R_{xx}(\tau + t_1 - t_2)dt_1dt_2$$

where $U$ and $\Lambda(t) = \begin{bmatrix} \cdot & e^{\lambda_nt} & \cdot \end{bmatrix}$ are the eigenvector and eigenvalue matrices respectively, being $\lambda_n$ the $n$-th eigenvalue, $R_{xx}(\tau)$ is the correlation matrix of the input excitation. If the dynamic loading is uncorrelated both in the time and in the space domain, then $R_{xx}$ is a constant diagonal matrix. This biasing constant is an unknown quantity in operational modal analysis because no input measurements are carried out. However it does not affect the estimate of the modal parameters.

2.2. Frequency domain approach, OFD

In the frequency domain approach, called Operational modal analysis in Frequency Domain (OFD), the odd part of the auto-power spectral density function could be obtained by Fourier transforming both sides of previous Eq. (1), [9], resulting in:

$$G_{y'y'}^o(f) = G_{y'y'}^e(f) * \left(-j\frac{1}{\pi f}\right) \quad (3)$$

where $f$ indicates the frequency (Hertz), whereas the asterisk the convolution operation. Because the right hand side is the Hilbert Transform of the even function $G_{y'y'}^e(f)$, then the previous equation could be rewritten so as to estimate the power spectral density function, $S_{y'y'}(f)$ of a causal signal, that is:

$$S_{y'y'}(f) = G_{y'y'}(f) - j\mathcal{H}\left[G_{y'y'}^e(f)\right] \quad (4)$$

When several measured channels are available, then causal cross power spectral density function could be obtained from Eq. (2), that could be rewritten as:

$$\Phi_{y'y'}(\tau) = \Phi_{y'y'}^e(\tau) + \Phi_{y'y'}^o(\tau)sgn(\tau) \quad (5)$$

and therefore, by performing the Fourier transform of both sides of the previous equation, one has:

$$S_{y'y'}(f) = G_{y'y'}^e(f) - j\mathcal{H}\left[G_{y'y'}^o(f)\right] \quad (6)$$

This relation is analogous to the one of Eq. (4). In this way, taking account of the part of the correlation function (both auto and cross) valid for time greater than zero, it is always possible to get
the spectral densities, that can be fitted also with the commercial codes. Indeed, collecting all the power spectral density functions of the output and input signals into the $S_{yy}(f)$ and $S_{xx}(f)$ matrices respectively, the modal model could be derived from the well known relation:

$$S_{yy}(f) = H(f)S_{xx}(f)H^H(f)$$

in which $H(f)$ is the frequency response function matrix and the superscript $^H$ is the Hermitian operator. In the case of white noise and uncorrelated in space input excitation the $S_{xx}(f)$ is a constant frequency independent diagonal matrix. This unknown biasing constant does not affect the estimate of the modal parameter as already seen in the time domain approach. Therefore the power spectral density can be fitted using a least square fitting - in the frequency domain - with a polynomial ratio, taking into account the part of the correlation function (both auto and cross) valid for time greater than zero.

3. VALIDATION OF OMA METHODS

Several tests have been carried out to validate both the OTD and the OFD approaches. Only the results from the test campaign carried out on a cantilever beam are reported here for the sake of brevity. The modal parameters estimated with the developed approaches are compared with those achieved with other OMA methods and with the traditional Input/Output based approach. The OMA methods considered for comparison are the Frequency Domain Decomposition (FDD) approach, [7], and the Balanced Realization (BR) method, [8], whereas the Input/Output approach (I/O) is the LMS-PolyMAX algorithm, [12]. This last approach is considered as the reference method for the purposes of the validation. The beam is randomly excited by tapping fingers, whereas an impulse excitation is used for the Input/Output estimates. The analysis was performed in the frequency range equal to [0-1024] Hz, the spectral lines were $2^{15}$ for an acquisition time of 16 s. From Tabs. 1 and 2, where the estimated natural frequencies and damping ratios are reported respectively, it is found a very good correlation of all the OMA approaches with the I/O reference one, also for the two proposed methods.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_n^{IO}$ [Hz]</th>
<th>$f_n^{FDD}$ [Hz]</th>
<th>$f_n^{BR}$ [Hz]</th>
<th>$f_n^{OFD}$ [Hz]</th>
<th>$f_n^{OTD}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>432.504</td>
<td>431.183</td>
<td>433.183</td>
<td>432.311</td>
<td>432.215</td>
</tr>
<tr>
<td>4</td>
<td>848.651</td>
<td>847.364</td>
<td>848.397</td>
<td>846.814</td>
<td>847.429</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\xi_n^{IO}$ [%]</th>
<th>$\xi_n^{FDD}$ [%]</th>
<th>$\xi_n^{BR}$ [%]</th>
<th>$\xi_n^{OFD}$ [%]</th>
<th>$\xi_n^{OTD}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.161</td>
<td>0.215</td>
<td>1.460</td>
<td>0.186</td>
<td>1.544</td>
</tr>
<tr>
<td>2</td>
<td>0.178</td>
<td>0.169</td>
<td>0.292</td>
<td>0.343</td>
<td>0.630</td>
</tr>
<tr>
<td>3</td>
<td>0.380</td>
<td>0.372</td>
<td>0.418</td>
<td>0.375</td>
<td>0.318</td>
</tr>
<tr>
<td>4</td>
<td>0.142</td>
<td>0.158</td>
<td>0.327</td>
<td>0.312</td>
<td>0.397</td>
</tr>
</tbody>
</table>

The differences in the natural frequency estimates are well below 1.5%, whereas although the damping ratios are characterized by much higher uncertainties, as expected, such differences are completely acceptable. Also the mode shapes are in a very good agreement. The mode shape correlations, evaluated by the MAC value do not fall below 90% among the modes, thus validating both the OTD and the OFD operational modal analysis approaches.
4. FLIGHT TESTS

The proposed OMA methods are used to identify the dynamic properties of a flying aircraft. The objective of such activity is the evaluation of the effectiveness of the both OTD and OFD approaches. The considered aircraft is a Savannah Vg 100hp, a high wing, single engine, ultra-light plane with side-by-side seating for two, Fig. 1a. The dynamic behaviour of the aircraft has been estimated from the acceleration responses recorded during the different flight phases by the LMS-SCADAS Mobile hardware, capable to simultaneously record up to eight channels (for this experimental setup) as well as the flown trajectory parameters thanks to the built-in GPS receiver. The measuring points where chosen in order to focus on the wing structure dynamics at different flight conditions. In Fig. 1b the location of the sensors is represented (the two upper right measuring points - referred as “origin1” and “origin2” - are reference points).

Among all the possible flying phases that could be analyzed from the flight tests, only the sensitivity to the flight speed, at a constant height, and the sensitivity to the height, at constant flight speed are presented in the paper. The considered flight conditions for the flight speed sensitivity are reported in Tab. 3 where, for each flight condition, the engine RPM, the indicated air speed (IAS) and the height are reported.

Table 3 Flight conditions for the flight speed sensitivity.

<table>
<thead>
<tr>
<th>Log number</th>
<th>Time stamp</th>
<th>Flight conditions</th>
<th>RPM</th>
<th>IAS [Km/h]</th>
<th>Height [ft]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>08:20-11:20</td>
<td>Horizontal flight</td>
<td>3200</td>
<td>100</td>
<td>1500</td>
</tr>
<tr>
<td>6</td>
<td>11:50-14:50</td>
<td>Horizontal flight</td>
<td>3500</td>
<td>120</td>
<td>1500</td>
</tr>
<tr>
<td>9</td>
<td>27:10-30:10</td>
<td>Horizontal flight</td>
<td>4200</td>
<td>140</td>
<td>1500</td>
</tr>
</tbody>
</table>

The developed approaches, OTD and OFD, estimated the modal parameters of the flying aircraft in a very good agreement with the other OMA approaches, that is FDD and BR. Moreover both the OTD and OFD methods were capable to detect changes of the system poles as function of the flight speed. In the root locus plot of Fig. 2 the sensitivity of the 10 modes, identified in the frequency range of

![Savannah ultra-light aircraft](image1)

![Flight speed sensitivity-root locus](image2)
interest and at an height of 1500 [ft] is reported. Specifically, the real and the imaginary part of the pole is reported in the xy axis respectively for each flight speed, that is here considered as the root-locus parameter. With the aim of having clear plots, only the average poles, obtained with the OTD and OFD, are used, because the difference between the two estimating techniques is very low and because such estimates are practically the same of the FDD and BR methods.

**Table 4 Flight conditions for the height sensitivity.**

<table>
<thead>
<tr>
<th>Log number</th>
<th>Time stamp</th>
<th>Flight conditions</th>
<th>RPM</th>
<th>IAS [km/h]</th>
<th>Height [ft]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>04:07-07:07</td>
<td>Horizontal flight</td>
<td>3900</td>
<td>120</td>
<td>1000</td>
</tr>
<tr>
<td>6</td>
<td>11:50-14:50</td>
<td>Horizontal flight</td>
<td>3500</td>
<td>120</td>
<td>1500</td>
</tr>
<tr>
<td>8</td>
<td>21:58-24:58</td>
<td>Horizontal flight</td>
<td>4000</td>
<td>120</td>
<td>2000</td>
</tr>
</tbody>
</table>

Further experimental activities investigated the capabilities of the proposed methods to track the evolution of the poles of the system as function of the height of flight. For this last case, the different flight conditions are reported in Tab. 4.

As in the previous aeroelastic analysis, the proposed approaches resulted capable to track the changes of the poles of the system as function of the flying height. Moreover the same poles as before were estimated with the same accuracy, and therefore the average poles, obtained by the OTD and OFD methods, are plotted in Fig. 3. for this case the flying height is considered as the root-locus parameter.

**Figure 3 Flight height sensitivity-root locus**

**Figure 4 Savannah's first and second aeroelastic mode shapes**

**Figure 5 Savannah's third and fourth aeroelastic mode shapes**
Finally, the experimental investigation highlighted that the identified mode shapes are not sensitive to the considered flight conditions. This means that the aircraft is tested in aeroelastic stable conditions. The identified mode shapes, for a given flight condition are reported from Fig. 4 to Fig. 6.

5. CONCLUSIONS

In this paper two novel approaches are proposed for the estimate of the modal parameters in the framework of the operational modal analysis. These methods operate both in the time (OTD) and in the frequency (OFD) domain and are based on the possibility to synthesize the analytical (causal) correlation or power spectral density matrix respectively from the actual time-limited response measurements. The properties of the Hilbert transform played a key-role in the theoretical developments because it allowed the estimate of the quadrature (conjugate) part of the signal. The approaches were experimentally validated against the more traditional Input/Output experimental modal analysis approach, as well as against other operational modal analysis methods. Finally the developed approaches were used estimate the aeroelastic behavior of the ultra-light aircraft Savannah from flight test data. This allowed to assess the accuracy of the approaches and demonstrated their capability to track the evolution of the poles of the system when varying the flight speed or the flight height. The proposed approaches could be considered for further developments concerning the problem of flutter clearance due to their promising numerical efficiency.

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