OPERATIONAL MODE SHAPE NORMALISATION OF SMALL AND LIGHT STRUCTURES

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ABSTRACT

A new method for normalisation of operational mode shapes was developed in last years and it is used mainly in construction engineering and on large structures. This method enables normalisation of mode shapes without the measurement of the excitation force and is called sensitivity-based operational mode-shape normalisation. The measured mode shapes are normalised by multiplying them with scaling factors, which are derived from the modal sensitivity of the structure on a change of the mass matrix. This procedure works with a presumption that the resonant frequencies shift, but mode shapes remain almost the same when the mass is added to the structure. The results are relatively good, when the added mass is well distributed over the structure and the amount is just right. But when measuring small and light structures (mass < 50 gram), the added mass can be too high and distributed unevenly. Consequently, not only the resonant frequencies shift, but also the mode shapes change. In this study an innovative method, which takes into account the mode shape variation, is presented. The calculation of normalised mode shapes is in this case based on the sensitivity analysis and structural modification of modal parameters. The numerical and experimental results of this method on small and light structures are compared with EMA and with ordinary sensitivity-based operational mode-shape normalisation. The advantages of the proposed method can be seen when measuring small and light structures, on which a variation of the mode shapes occurs due to the added mass of the sensor.

Keywords: Operational Modal Analysis, Normalisation, Structural modification, Small and light structures

1. INTRODUCTION

Small and light structures (mass < 50 gram) have some distinctive features, which cause difficulties in the measurement of their modal parameters. The major issues are the mass, which is added to the measured structure by sensors, and the very high resonant frequencies. Usually the experimental modal analysis (EMA) is used to define the modal parameters of small and light structures. Different methods for EMA are described in [1], where an innovative method with a custom-made force sensor is proposed. A very light force sensor (0.4 gram added mass) with wide frequency range of

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measurement (up to 20 kHz) is used. The method, used by Rovšček et al. in [1] is also used as a comparison to operational modal analysis (OMA) in this study. It is more thoroughly described in Section 4.1.

OMA was rarely used on small and light structures in the past, especially because the operational mode shapes could not be mass normalised and therefore the modal description of the structure was not complete [2]. But in 2002 a new method was introduced by Parloo et al. [3], which makes the mass normalisation of the operational mode shapes possible. The operational mode shapes are normalised by multiplying them by scaling factors, which are derived from the modal sensitivity of the structure on a change of the mass matrix. Therefore, the term mass-change strategy is frequently used to denote Parloo's method. The possibility and difficulties of using the mass-change strategy on small and light structures have not yet been analysed in detail.

The mass-change strategy works with the presumption that the resonant frequencies shift, but the mode shapes remain almost the same when the mass is added to the structure, as shown by Parloo et al. [3]. The results are relatively good, when the quantity of the added mass is just right (about 5% of the whole mass of the structure), as Lopez-Aenlle et al. [4] concluded. Furthermore, the added mass needs to be well distributed over the structure. But when measuring small and light structures, the added mass can be too high and not well distributed. For this reason, not only do the resonant frequencies shift, but the mode shapes also change.

In this study an innovative method that takes into account the mode-shape variation due to accelerometer added mass is presented. The normalisation of the operational mode shapes is based on a sensitivity analysis and a structural modification of the modal parameters. The results of the proposed operational mode-shape normalisation method are compared to the results of the EMA procedure presented by Rovšček et al. in [1] and to the numerical model. The method proposed in this study gives significantly better results than the EMA, especially at low frequencies (under 500 Hz), and is simpler to use, since the measurement of the excitation force is not needed. It can be used on any similar small and light structure. The main advantage of this method compared to the sensitivity-based operational mode-shape normalisation proposed by Parloo et al. [3] is that it also takes into account the mode-shape variation, which occurs due to the added mass of the accelerometer.

2. THEORETICAL BACKGROUND

2.1. Sensitivity-based operational mode-shape normalisation

The operational mode shapes are not mass normalised, since the excitation force is not measured when performing the OMA. Therefore, a new approach was proposed by Parloo et al. [3] and later more thoroughly analysed by Lopez-Aenlle et al. [4] to normalise the operational mode shapes. The main idea of this approach is to mass normalise the mode shapes by multiplying them with scaling factors. The scaling factors are calculated on the basis of the modal sensitivity of the structure. By adding a known mass to the selected points of the structure the resonant frequencies shift. From these shifts the scaling factors for each mode shape can be calculated using the sensitivity analysis. The term mass-change strategy is frequently used to denote this method.

A known mass \([\Delta M]\) needs to be added to the structure to perform the mass-change strategy. The change of the mass matrix causes a variation of the modal parameters. If the modal parameters of the unmodified and modified structure are measured, then the scaling factors \(\alpha_i\) can be calculated, as shown in Equation (1).

\[
\alpha_i = \sqrt{\frac{(\omega_i^2 - \omega_{i,M}^2)}{\omega_{i,M}^2 \left[ \psi_i \right]^T [\Delta M] \left[ \psi_i \right]}}
\]

where \(\omega_i\) denotes the \(i\)-th resonant frequency of the unmodified structure and \(\omega_{i,M}\) is the \(i\)-th resonant frequency of the modified structure (when the mass \([\Delta M]\) is added). \([\psi_i]\) is the unnormalised mode
shape of the structure. A detailed derivation of Equation (1) can be found in [4], where a presumption is made that the mode shapes do not change significantly when adding the mass to the structure \( \{ \psi_i \} \approx \{ \psi_i, M \} \). The mass-normalised mode shapes \( \{ \phi_i \} \) can then be obtained by multiplying the unnormalised mode shapes \( \{ \psi_i \} \) by the scaling factors \( \alpha_i \):

\[
\{ \phi_i \} = \alpha_i \{ \psi_i \}
\]

(2)

2.2. Operational mode-shape normalisation of small and light structures

The mass-change strategy with the scaling factors shown in Equation (1) gives relatively good results for most of the structures. But when small and light structures are analysed (Figure 1), the mass of the accelerometer can be relatively large in comparison to the other masses added to the structure for the normalisation [6]. Therefore, the added mass is not well distributed over the structure and not only do the resonant frequencies change, but also the mode shapes. The condition \( \{ \psi_i \} \approx \{ \psi_i, M \} \) is not fulfilled, which leads to incorrect scaling factors \( \alpha_i \) (Figure 1).

![Figure 1: Comparison of the mass-change strategy and the method used in this study](image)

To correctly measure and normalise the mode shapes of the structure itself an innovative method is proposed in this study. The mass-change strategy is performed on the structure with the accelerometer. The added mass is well distributed over the modified structure; therefore, the mode shapes of the unmodified and modified structure do not differ significantly (the condition \( \{ \psi_i \} \approx \{ \psi_i, M \} \) is fulfilled). Consequently, the scaling factors \( \alpha_i \) are correct and the mode shapes of the structure with the accelerometer can be normalised. Then the variation of the mode shapes due to the added mass of the accelerometer needs to be determined and the normalised mode shapes \( \{ \phi_i^* \} \) of the structure itself (without the accelerometer) can be obtained. A proper method is needed to determine the mode-shape variation due to the accelerometer’s added mass. The method used in this study is described in Section 2.3.

2.3. Variation of the modal parameters due to a structural modification

The variation of the modal parameters due to the structural modification was calculated with a method described by Chen [5]. Chen proposed an iterative procedure to provide exact predictions for the resonant frequencies and the corresponding mode shapes of the modified structure. Chen’s iterative procedure gives approximations of the \( i \)-th modified eigenvalue \( \lambda_i^* \) and eigenvector \( \{ \phi_i^* \} \), which are determined on the basis of the \( i \)-th unmodified eigenvalue \( \lambda_i \) and eigenvector \( \{ \phi_i \} \) of the structure. The procedure is presented in Equations (3) and (4).
\[
\{ \phi_i^* \} = \{ \phi_i \} + \sum_{k=1, k \neq i}^{N} C_{ik} \{ \phi_k \} 
\]  
\( (3) \)

\[
\lambda_i^* = \lambda_i + \Delta \lambda_i 
\]  
\( (4) \)

where: \( \lambda_i = \sqrt{\omega_i^2 (1 + \eta_i)} \)

The \( i \)-th eigenvalue \( \lambda_i \) contains the information on the resonant frequency \( \omega_i \) and the damping loss factor \( \eta_i \); the eigenvector \( \{ \phi_i \} \) represents the \( i \)-th normalised mode shape. The modification of the \( i \)-th eigenvalue is denoted by \( \Delta \lambda_i \) and the modification of the \( i \)-th eigenvector can be expressed as a linear combination of all the independent original eigenvectors except for the corresponding one (\( k \neq i \)). \( C_{ik} \) is a participation factor of the \( k \)-th eigenvector when calculating the modification of the \( i \)-th eigenvector. The definitions of the modifications of the eigenvalues \( \Delta \lambda_i \) and the participation factors \( C_{ik} \) are presented in Equations (5) and (6), derived by Chen [5], where \( a_{ii}^K \) and \( a_{ii}^M \) are defined as shown in Equations (7) and (8).

\[
\Delta \lambda_i = \frac{ \left( a_{ii}^K - \lambda_i a_{ii}^M \right) + \sum_{k=1, k \neq i}^{N} \left( a_{ii}^K - \lambda_i a_{ii}^M \right) C_{ik} }{ 1 + a_{ii}^M + \sum_{k=1, k \neq i}^{N} a_{ii}^M C_{ik} } 
\]  
\( (5) \)

\[
C_{ik} = \frac{ \left( a_{ii}^K - \lambda_i a_{ii}^M \right) + \sum_{l=1, l \neq i, k}^{N} \left( a_{ii}^K - \lambda_l a_{ii}^M \right) C_{il} }{ \left( \lambda_i^* - \lambda_k \right) - \left( a_{kk}^K - \lambda_k a_{kk}^M \right) } 
\]  
\( (6) \)

\[
a_{ii}^K = \phi_i^T \Delta K \phi_i 
\]  
\( (7) \)

\[
a_{ii}^M = \phi_i^T \Delta M \phi_i 
\]  
\( (8) \)

It is clear from Equations (5) and (6) that the calculations of \( \Delta \lambda_i \) and \( C_{ik} \) do not require a knowledge of the mass and stiffness matrices of the original or modified structures. It is sufficient to know the modification of these two matrices (\( \Delta M \) and \( \Delta K \)). The whole iterative procedure is done by repeatedly alternating between (5) and (6) until the calculated values are accurate enough. In the first approximation of \( \Delta \lambda_i \) we use the value \( C_{ik} = 0 \) for all \( i \) and \( k \). The described iterative procedure of Chen was used in this study to determine the mode-shape variation caused by the additional mass of the accelerometer.

### 3. SAMPLE

To compare the OMA and the numerical model, a sample of simple geometry with the proper geometrical and modal characteristics was needed. A steel sample of circular cross-section with 6-mm diameter, 108-mm length and 21-gram mass was used, as shown in Figure 2. Its geometry was chosen in such a way as to ensure one axial-mode, one torsional-mode, and many bending-mode shapes in the frequency range of the measurement (0-20 kHz). In this study the focus will only be on the first five bending-mode shapes in the direction of the \( Y \) axis. Other mode shapes and resonant frequencies can be measured in the same manner. Fifteen points on the structure, which are shown in Figure 2, were used for the measurements; therefore, the results of the model will also be calculated for these points. The sample that was used for this study is a homogenous structure without any joints. It has well-
known material characteristics and geometry; therefore, a relatively accurate FEM was formed based on this.

![Sample under investigation](image)

**Figure 2** Sample under investigation

4. EXPERIMENTAL INVESTIGATION

The aim of the experimental investigation was to analyse the advantages and disadvantages of the proposed method for the normalisation of the operational mode shapes in comparison to the mass-change strategy (as demonstrated by Parloo et al. [3] and Lopez-Aenlle et al. [4]) and also to classic EMA [2] with a known excitation, because EMA is the most frequently used method.

4.1. EMA

Measurements of the FRFs were performed by monitoring the excitation force and the response (velocity) of the structure simultaneously. The sample and the LDS V-101 electromagnetic shaker (excitation device) were suspended by strings to simulate the free-free support, as shown in Figure 3. The measurement of the response was performed using a LDV Polytec PDV-100 at fifteen points (1-15) in the Y direction, as shown in Section 3 (Figure 2). The measurement points were the same for the OMA. The structure was excited at point 9 for the EMA.

![Experimental setup for the EMA](image)

**Figure 3** Experimental setup for the EMA

A custom-made sensor was developed to measure the excitation force and transfer it from the shaker to the sample. This sensor is based on a piezo strain gauge (PCB 740B02). Its structure and functioning are described in an article by Rovšček et al. [1], where a detailed description of the EMA procedure can also be found. The effect of the added mass of the force sensor on the structure was small (0.4 gram) and neglected in the calculation of the modal parameters.

4.2. OMA

The sample was suspended by two strings when performing the OMA, as shown in Figure 4. The ambient excitation was carried out using a small steel ball (4 mm diameter) that was glued to a string and swung into the structure to achieve impact excitation. The steel ball hit the structure in point 9.
where all the measured modes were excited well. The reference response was measured by a B&K 4517-002 accelerometer, which was positioned at point 15. The response at the \( i \)-th point on the structure \( (i = 1, 2, 3, \ldots, 15) \) was measured using a LDV Polytec PDV-100. The resonant frequencies \( \omega_i \) and unnormalised mode shapes \( \{\psi_i\} \) were extracted from the response measurements. For the normalisation of the mode shapes with the mass-change strategy, eight magnets, each with a mass of 0.21 gram, were added at points 1, 3, 5, 7, 9, 11, 13 and 15 on the structure and the resonant frequencies \( \omega_{i,M} \) of the changed structure were measured. Then the scaling factors \( \alpha_i \) were calculated, as shown in Equation (1), to normalise the operational mode shapes of the structure with the accelerometer at point 15 (load case B). The mode shapes of the structure itself (without the accelerometer - load case B*) were obtained by using the method described in Section 2.3.

The accelerometer was deliberately placed at the point 15 to achieve sufficient modification of the mode shapes to clearly present the advantages of the proposed method. Point 15 is otherwise not the best position for this sensor to obtain good results with the OMA.

5. COMPARISON OF THE RESULTS

The modal parameters were determined for the structure without the added mass (load case A) and for the structure with the added mass of the accelerometer at point 15 (load case B). Since the damping is based on the results of the measurement, it is reasonable to compare the resonant frequencies and mode shapes only. The mode shapes are denoted with indexes representing modal analysis method (num, EMA, OMA) and load case (A - no added mass, B - accelerometer added mass, B* - cancelled load of the accelerometer). The correlation of the mode shapes was calculated using the Modal Assurance Criterion (MAC).

5.1. Resonant frequencies

The comparison of the resonant frequencies calculated with the numerical model and measured with the EMA and OMA is presented in Table 1. The resonant frequencies for OMA-A (where A denotes the load case) were measured only with the LDV (without the added mass of the accelerometer). All the other OMA measurements were performed with the accelerometer and the LDV, as described in Section 4.2.

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It is clear from Table 1 that the resonant frequencies of the structure without the accelerometer (num-A, EMA and OMA-A) differ by less than 2%. The resonant frequencies of the structure with the accelerometer at point 15 (num-B, OMA-B) are even closer together (less than 0.7% difference). Therefore, the measured values of the resonant frequencies are accurate in both cases, EMA or OMA.
5.2. Mode shapes

The results of the OMA were compared with those of the numerical model. The mode shapes were measured for load case B (with the added mass of the accelerometer), since this measurement for A is not possible when using an accelerometer. The comparison of the measured (OMA) and the calculated mode shapes is presented in Figure 5 and Figure 6.

![Figure 5 Numerical (num-A) and operational (OMA-B, OMA-B*) mode shapes.](image1)

![Figure 6 MAC correlation between numerical (num-A) and operational (OMA-B and OMA-B*) mode shapes.](image2)

In Figure 5 it is clear that the scale of the numerical and OMA mode shapes are similar. The similarity of the numerical and OMA mode shapes confirms that the scale factors $\alpha_i$ were calculated correctly. If the numerical load case A (without the added mass) is compared to the OMA measurement results (load case B) the correlation is not so good, as shown in Figure 6 (left). If the variation of the modal parameters because of the structural modification is taken into account, then the mode shapes OMA-B transform into OMA-B*. Figure 6 (right) shows that OMA-B* is well correlated with num-A (all the diagonal MAC values are above 0.71; the first three even above 0.95). This proves that the procedure proposed in this study can be used to measure the mass-normalised mode shapes of small and light structures, even if the variation of the mode shapes occurs during the measurement. This variation is usually caused by the added mass of the sensors.

![Figure 7 MAC correlation between experimental (EMA) and operational (OMA-B and OMA-B*) mode shapes.](image3)
When the results of the OMA are compared with the results of the EMA, as shown in Figure 7, it is clear that a better correlation of the mode shapes is achieved if the variation of the modal parameters is taken into account. The results show that the EMA mode shapes in the frequency range of the force sensor (500 Hz - 20 kHz) are well correlated with those of the OMA. The first mode shape is below 500 Hz; therefore, the correlation is not as good as for the modes 2-5. Better correlation of OMA-B* and EMA in comparison to OMA-B and EMA confirms that the consideration of the accelerometer's added mass significantly improves the results.

6. CONCLUSION

This study presents an innovative procedure for the normalisation of the operational mode shapes of small and light structures. The operational mode shapes are normalised by a sensitivity-based method. The proposed procedure takes into account the mode-shape variation that occurs because the mass is unevenly distributed when performing a mass-change strategy. The uneven distribution of the added mass often occurs on small and light structures, where the effects of the sensor's added mass are larger than usual.

The procedure presented in this study is tested on a sample of small dimension and mass. The measured modal parameters are compared to the numerical model and between EMA and OMA. The comparison proves that the proposed procedure gives better results than the ordinary mass-change strategy, because the mode shape variation due to the sensor's mass load is taken into account. The mode shapes are measured and scaled correctly, which makes the proposed method suitable for modal analysis of small and light structures. It is a good alternative to EMA methods and makes the modal analysis more straightforward, because the measurement of the excitation force is not needed.

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