ORDER-BASED MODAL ANALYSIS: A MODAL PARAMETER ESTIMATION TECHNIQUE FOR ROTATING MACHINERIES

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ABSTRACT

Rotating machineries are employed today in a wide variety of industrial applications. There are few machines and other mechanical systems which do not include rotating components such as wheels, gears, bearings, shafts and seals. Turbines including steam, gas and wind, electrical motors and generators, turbochargers, and internal combustion engines, are just some examples of rotating machines.

In the presence of rotating machineries, the so-called orders appear as multiples or fractions of the rotational speed. They can be defined as the proportional constants between the rotational speed and the frequency. The Order Tracking (OT) theory estimates the amplitude and the phase of the orders. It can be stated that during a run-up or run-down test, the measured response is mainly caused by rotational excitation. This is the main reason behind the idea to perform Operational Modal Analysis (OMA) on tracked orders rather than on the overall spectrum.

It is assumed that the structure is excited by a rotating mass with increasing frequency. The excitation can be represented by two perpendicular forces of equal amplitude and in quadrature. The method considers the run-up as a multi-sine sweep excitation and combines advanced order tracking techniques with operational modal analysis to identify the modal parameters. The procedure is known as Order-Based Modal Analysis (OBMA).

Keywords: Order tracking, Modal Analysis, Rotating machineries, Wind turbine gearbox
1. INTRODUCTION

Rotating machineries face several complex and non-linear conditions during their operating life. Several techniques have been developed to perform a dynamic characterization of such machines, but the problem remains quite challenging. It can be distinguished among techniques that try to characterize the dynamic properties of the machine under study and techniques that attempt to identify the response signals measured while the machine is operating. Modal analysis is included in the first category, while the Order tracking is part of the second group of techniques.

In the past years Operational Modal Analysis (OMA) has become the most commonly used method to derive an experimental modal model from vibration measurements in operational conditions. This method is based on several assumptions which are not always fulfilled in case of rotating machineries. This is the main reason why new techniques need to be explored. Combining together the two families of techniques, the so-called Order-Based Modal Analysis (OBMA) has been developed [1]. It can be seen as a combination of existing Order tracking and Operational Modal Analysis and it is able to identify resonances of the structure from operational data during a run-up/run-down by considering it as a multi-sine sweep excitation in the frequency band of interests.

2. THEORETICAL BACKGROUND

2.1. Order tracking

Order tracking is the analysis of frequency components whose frequency is related to the rotational frequency of the operating machine. If the machine is running in non-stationary conditions, then the frequency components will be time varying and some more information are needed in order to perform the analysis. The additional information are in the form of tachometer signals measured on reference shafts of the machine. Several methods have been employed to digitally track orders which results from rotating components in noise and vibration problems [2]. In order to allow the computation of the exact frequency possessed by the order of interest, an accurate tachometer signal is needed for all the methods:

- Time domain sampling based Fast Fourier Transform (FFT) order tracking;
- Angle domain computed order tracking;
- Time Variant Discrete Fourier Transform (TVDFT);
- Vold-Kalman (VK) filter based order tracking.

The last two methods will be analyzed with more details since it has been demonstrated that they are the ones which give the best results in terms of tracked orders

2.1.1. Time Variant Discrete Fourier Transform (TVDFT)

The Time Variant Discrete Fourier Transform (TVDFT) method gives results very similar to the resampling based order tracking, but with less computational efforts. It is based on a Fourier transform kernel whose frequency is allowed to vary with time and it does not require the transformation from the time domain to the angle domain. The TVDFT is based on kernels in which the sine and cosine functions have unity amplitude and an instantaneous frequency matching that of the tracked order at each instant in time [3].

The formulation can be extended in order to separate close or crossing orders through a secondary calculation. There can be a leakage error using the TVDFT with constant Δt sampled data because it is not guaranteed that the integer revolution values required for a constant order bandwidth analysis will fall on a Δt. If it is not the case, it will lead to a leakage error by performing the transformation over a non-integer number of revolutions. This error can be reduced by oversampling the data which means by providing a finer Δt. This method contains most of the advantages of the resampling based order tracking and it can be implemented in a very efficient manner without having the computational load and complexity of the transformation from the time domain to the angle domain.
2.1.2. **Vold-Kalman (VK) filter based order tracking**

Vold and Leuridan introduced an algorithm for high resolution, slew rate independent order tracking based on the concepts of Kalman filter. The Vold-Kalman (VK) algorithm allows tracking multiple orders at the same time and it is able to decouple close and crossing orders. This method extracts the time history of the order as well as the estimate of the amplitude and the phase of the same order [4].

Similarly as the Kalman filter is based on the process and measurement equations [5], the VK filter is based on the structural and data equations. The structural equation is an equation that describes the mathematical characteristics of the order to be extracted. It uses the information from the tachometer signal and it describes a sine wave whose frequency and amplitude is constant over three consecutive time points. In order to take into account the variations from a perfect sine wave over the time samples involved, the unknown non-homogeneity term \( \varepsilon(n) \) is introduced on the right side of Eq. (1).

\[
x(n) - 2\cos(\omega \Delta t) x(n-1) + x(n-2) = \varepsilon(n)
\]

where \( x(n) \) represents the \( n \)-th discrete time sample and \( \omega \) is the instantaneous frequency of the sine wave.

The second equation on which the Vold-Kalman filter is based on is the so-called data equation and it is shown in Eq. (2). It describes the relationship between the order \( x(n) \) and the measured data \( y(n) \). Normally the measured data is a combination of all the orders generated from the machine and other random noise present in the data. The random noise and non-tracked orders are combined into the signal \( \eta(n) \).

\[
y(n) = x(n) + \eta(n)
\]

A weighted solution can be obtained by introducing the Harmonic Confidence Factor (HCF) \( r \). The value of this parameter is what determines the tracking characteristics of the filter. It is calculated, as shown in Eq. (3) as the ratio between the standard deviations of the structure and data equations.

\[
r(n) = \frac{s_x(n)}{s_\eta(n)}
\]

The choice of a large value for the weighting factor \( r \) leads to a highly selective filtration in the frequency domain, while by choosing \( r \) small the resolution in frequency is very small, but a fast convergence in amplitude can be obtained. Applying the ratio as a weighting function and combining together the two equations, the set of linear equations shown in Eq. (4) is obtained.

\[
\begin{bmatrix}
1 & -2\cos(\omega \Delta t) & 1 \\
0 & 0 & r(n)
\end{bmatrix}
\begin{bmatrix}
x(n-2) \\
x(n-1) \\
x(n)
\end{bmatrix}
= \begin{bmatrix}
\varepsilon(n) \\
r(n)(y(n) - \eta(n))
\end{bmatrix}
\]

Applying Eq. (4) to all observed time points will give a global system of over determined equations for the desired waveform \( x(n) \) which may be solved by using standard least squares techniques such as normal equations or the singular value decomposition. For the purpose of order tracking, the filtered waveform is most conveniently described in terms of amplitude and phase with respect to a reference channel such as the tachometer channel.

### 2.2. Operational Modal Analysis

The Operational Modal Analysis (OMA) technique allows the identification of the modal parameters by using operational measurements such as accelerations measured on several points on the structure. This is the reason why it is also known as output-only modal analysis. Several assumptions need to be fulfilled in order to apply successfully the technique: the system must be linear time invariant and the excitation forces have to be represented by a flat white noise spectrum in the frequency band of interest. The main difference with the classical modal analysis technique is due to the use of auto- and
cross-correlations and auto- and cross-powers in place of impulse and frequency responses. Some reference signals need to be considered to perform the identification procedure.

The Polymax method, also known as the polyreference least-squares complex frequency-domain method, has been introduced as a new standard for modal parameter estimation by Peeters et al. [6]. It is a technique based on a weighted least-squares approach by using multiple-input-multiple-output frequency response functions as primary data. First a stabilisation diagram can be constructed and frequencies, damping and participation information can be extracted. Afterwards the mode shapes are found in a second least-squares step, based on the selection of the stable poles. The main advantage of such a method if compared to other methods is the very clear stabilisation diagram that can be obtained allowing simple stable poles identification.

The Polymax method has been extended for performing Operational Modal Analysis [7]. In this case it requires output spectra as primary data. Under the assumption of white noise excitation, output spectra can be modelled in a very similar way as Frequency Response Functions (FRFs).

2.3. Order-Based Modal Analysis

Several works have demonstrated that the results of the spectrum-based approach look very satisfactory, but they have to be interpreted with care. Some of the peaks in the overall spectrum are originated from order components which suddenly stop at the maximum rpm. These “end-of-order” peaks are identified as poles of the system even if they are not physically in the system [8]. A new technique has been developed which does not suffer of the “end-of-order” effect. The order tracking techniques introduced in Section 2.1 are combined together with the Operational Polymax introduced in Section 2.2 to identify the resonances from the orders instead than from the spectra.

The idea of performing OMA based on tracked orders instead of considering the overall spectrum arose because, during a run-up or run-down test, the measured response are mainly caused by the rotational excitation. In this formulation, the run-up or run-down is then considered as a multi-sine sweep excitation in the frequency band of interest. The excitation force acting on the structure can be considered as a rotating mass with increasing (or decreasing) frequency which can be represented by two correlated perpendicular forces of equal amplitude and in quadrature (90° phase difference).

Modal analysis can be applied to displacement orders taking into account the listed observations:

- Displacement orders are proportional to the squared rotation speed and, as consequence, acceleration orders are proportional to the forth power of the same rotation speed. The main difference is that in the classical modal analysis the acceleration FRFs are proportional to the squared frequency axis.
- Complex upper and lower residuals, while in classical modal analysis they are real.
- Complex participation factor both in classical modal analysis and in order-based modal analysis.

Methods such as Operational Polymax are robust against these observations and they can be employed for estimating the modal parameters in case of rotating machineries by looking at the orders rather than at the spectra.

3. NUMERICAL EXAMPLE

An analytical example has been built with the main purpose to validate the proposed procedure and to compare the different methods in order to identify the best combination of Order tracking and Operational Modal Analysis techniques.

The model is an 8 degree of freedom (DOF) mass-spring-damper planar system which is composed of 4 masses connected with springs and dampers. The directions x and y are coupled and a schematic representation is shown in Figure 1. A rotating mass excitation has been applied on mass m2 to simulate the effect of a rotating force which can be represented by two perpendicular forces of equal amplitude, but in quadrature as shown in Eq. (5).
In order to simulate a run-up, the response of the structure has been recorded while the frequency of the rotating force was increasing from 0 to 50 Hz over 120 seconds.

As first step, the FRFs have been calculated by using as references the two forces \( f_x(t) \) and \( f_y(t) \) and Polymax method has been applied to estimate the natural frequencies and damping ratios. These values are listed in Table 1 and will be used as reference for the implemented order tracking methods since they are corresponding exactly with the ones obtained from the system matrices of the numerical model [9].

\[
\begin{align*}
    f_x(t) &= rm\omega_0^2\cos(\omega_0 t + \phi) \\
    f_y(t) &= rm\omega_0^2\sin(\omega_0 t + \phi)
\end{align*}
\]

In the case of a sine sweep, the excitation can be considered as a broad-band white noise in the frequency range of the sweep. This is the reason why Operational Modal Analysis can be performed by processing the acceleration data. Cross power spectra can be calculated by using as reference the accelerations on mass 2 in the two direction x and y. However, by applying OMA on the overall spectra the so-called “end-of-order” effect can arise if an order is ending in the considered frequency band. The system under study is not affected by this problem because the excitation order ends at 50 Hz and all the modes of interest are below this frequency.

Order-Based Modal Analysis (OBMA) can be applied. The orders have to be extracted from the data and the rotational speed is needed in order to apply the order tracking methods proposed in Section 2. Another important point to underline is that OBMA requires a reference signal related to the excitation for calculating the phase. A sine sweep with frequency equal to the instantaneous rotational speed has been chosen as reference signal for phase calculation. Examples of an extracted order in a reduced frequency band (10 Hz – 28 Hz) by using the two order tracking methods is shown in Figure 2. In this frequency band, three modes have been identified, as can be seen from Table 1. The identified orders are compared to the theoretical order.

![Figure 1. Schematic representation of the analytical model.](image)

![Figure 2. Comparison of orders extracted by using the TVDFT technique (left) and the VK technique (right) with the theoretical order.](image)
For what concerns the Vold-Kalman filter based order tracking procedure, the main problem in terms of input values is related to the $r$ value which allows modifying the selectivity of the filter in the frequency domain. This value cannot be too large because otherwise the convergence would be reached in a very long time, but it cannot be very small as well because the resolution in frequency would be very small. A stabilization diagram by using the orders obtained with the Vold-Kalman order tracking technique is shown in Figure 3.

![Stabilization diagram](image)

**Figure 3.** Stabilization diagram obtained by using the Vold-Kalman order tracking technique.

In the numerical case, the two methods perform quite well. There are some differences in terms of resolution which is higher in the VK case. If compared with the FFT based and the angle domain methods, some improvements are clearly seen when the Time Variant Discrete Fourier (TVDFT) or the Vold-Kalman filter order tracking techniques are applied. The FFT based and the angle domain methods identify the orders less smoothly both in phase and in amplitude. In case of non-steady state conditions, all the methods have some difficulties to identify the instantaneous amplitude. For this reason low damped modes show up in the tracked order spectra at a higher frequency and with an increased damping. The results are summarized in Table 1.

<table>
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<tr>
<th>Frequency [Hz]</th>
<th>Damping [%]</th>
<th>Frequency [Hz]</th>
<th>Damping [%]</th>
<th>Frequency [Hz]</th>
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</tr>
</tbody>
</table>

### Table 1. Order-Based Modal Analysis results by considering the two different order tracking techniques compared to the numerical parameters of the system.

4. **WIND TURBINE GEARBOX TEST CASE**

Before being installed in a wind turbine, gearboxes need to be tested under representative loading conditions using parameterized load cases. Potential technical risk can be identified well in advance by accelerating the life testing. The final aim of these tests is to improve gearbox reliability which is a very actual topic of discussions in the wind turbine domain.

The measurement campaign took place on the 13.2 MW dynamic test rig available at ZF Wind Power. A 3.2 MW and a 3 MW prototype were placed in a back-to-back configuration with one gearbox (P3)
in “generator mode” as in the wind turbine and the other (P2) in “motor mode”. Since they have a slightly different gear ratio, while the P3 gearbox is being tested, the P2 is not running at nominal speed. Several conditions have been performed (constant speed, run-up, shaker measurements) and over 250 points were measured by means of tri-axial accelerometers. Due to the limited amount of available measurement channels, 7 batches for each configuration needed to be done by moving all the accelerometers while few of them were fixed in the same locations along all the measurement campaign to allow data merging for a more efficient processing. All the operational conditions were performed at different torque level (33%, 66% and 100% of the nominal torque value). In this case, only the 100% data have been analysed.

In this case, by applying the OMA on the overall spectra, the mentioned “end-of-order” can be easily identified because several orders are ending in the considered frequency band [10]. In a typical gearbox several orders are present in the signals, due to the rotation speed of the two shafts (high speed shaft and low speed shaft) and due to the several gear mesh frequencies which appear as non-integer multiples of the fundamental order. Moreover, in the test rig configuration, as mentioned, the two gearboxes have a slightly different gear ratio which doubles the number of orders. This means that there will be several orders ending in the selected frequency band (0 Hz – 750 Hz).

Finally, orders have been extracted to apply Order-Based Modal Analysis. During the measurement campaign, three different tachometer signals have been measured. One optical sensor was placed on each High Speed Shaft (HSS) and one on the Low Speed Shaft (LSS). However, because of the noise, only one of the signals was useful in the analysis to extract correctly the orders.

Orders were calculated using the Angle domain order tracking method, the TVDFT and the Vold-Kalman ones. A proper phase reference is obtained in three steps. First of all the tachometer signal is multiplied by the order to be extracted, then the instantaneous angle is calculated and finally the corresponding synchronous sine wave can be extracted.

The results obtained applying the Order-Based Modal Analysis technique on one of the dominant order look quite satisfactory in terms of natural frequencies. Nice stable poles columns are obtained and they can be easily identified. Even if the stabilization diagram is quite clear, the noise in the orders makes the synthesis very poor and the mode shapes noisy and non-consistent.

![Figure 4](image)

**Figure 4.** Signature of the gearbox on the test rig during the run-up (left); comparison between orders extracted with the two different techniques (right).

By looking at Figure 4 is quite clear that the two algorithms have comparable performances. The Vold-Kalman filter order tracking has the advantage to have the same resolution in the complete frequency band, while the resolution of the order extracted by using the TVDFT algorithm improves when the frequency increases.

5. **CONCLUSIONS**

The problem of estimating the mode shapes by considering the orders instead of the overall spectra has been investigated in more details. The main advantage of combining together Order tracking and Operational Modal Analysis is the fact that the so-called “end-of-order” effect is not present in the data. Several order tracking techniques have been implemented and tested in several cases. The Vold-
Kalman filter order tracking has been proven to be a very promising technique in terms of combination with Operational Polymax. More investigations will be done in the future, whereas in this paper the main focus has been on two different test cases: a numerical one in which the techniques have been validated and an experimental one where several more problems needed to be faced. In future works the Order-Based Modal Analysis (OBMA) method will be tested in different situations in order to better understand its limitations and drawbacks.

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