OPERATIONAL MODAL ANALYSIS AND DETECTION OF NON-LINEAR STRUCTURAL BEHAVIOR OF BOWSTRING ARCH BRIDGE

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ABSTRACT

Most Structural Health Monitoring (SHM) techniques are based on the variation of modal properties of structures. Modal analysis is established under the assumption that the structure behaviour is linear. However, the presence of non-linearities (local or global, smooth or strong) can affect the dynamic properties of the structure. Most of techniques to detect non-linear behaviour in structural dynamics are based on frequency domain data. Fewer techniques only deal with the use of time responses. The main aim of the present research is to check the suitability of Principal Component Analysis (PCA) technique in a real case: The Palma del Río bowstring arch bridge. Within an assessment project developed between the Universities of Córdoba and Granada with INECO, the state of this bridge was evaluated by an ambient vibration test (AVT) on June 2014

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1. INTRODUCTION

Structural Health Monitoring (SHM) techniques are considered the most effective method to quantify the actual state and serviceability of big scale structures in service. These techniques employ field testing of structures for different purposes: system identification, structural damage identification, and finite element updating \cite{1,2,3,4,5}. Conventional SHM techniques are developed based on linear elastic theory. However, cases of non-linearities are quite common in practice. Some of the sources of this non-linearities are cracks and damages. Structural damage identification techniques take advantage of this transition from linear to
nonlinear response in the damage-detection process.

Non-linear detection in big scale structures is still a challenging issue for engineers. Some difficulties to measure the response of a non-linear system are due, partly because the most measurements are taken on non-linear structures with a small number of degrees of freedom, or the response is considered as a perturbation to the linear response. Ritto [6] recently considered the application of the effective independence method to the proper orthogonal modes of a non-linear structure; this paper investigates the properties of this approach in its application in a real case.

2. PROPER ORTHOGONAL DECOMPOSITION (POD)

Suppose that the response of a structure is measured at \( n \) degrees of freedom and at \( m \) samples. Thus \( x_i(t_j) \) represents the measurement of the \( i \)th degree of freedom at the \( j \)th time instant. This time response history is assembled into a matrix given by

\[
X = \begin{bmatrix}
x_1(t_1) & \cdots & x_1(t_n) \\
\vdots & \ddots & \vdots \\
x_n(t_1) & \cdots & x_n(t_m)
\end{bmatrix}.
\]

The singular value decomposition (SVD) of the matrix \( X \) is then calculated as

\[
X = USV
\]

where \( U \) and \( V \) are orthogonal matrices of dimensions \( n \times n \) and \( m \times m \) respectively, and \( S \) is an \( n \times m \) matrix where only the diagonal terms are non-zero. The columns of \( U \) are called the Proper Orthogonal Modes (POMs) and the diagonal elements of \( S \) are the corresponding Proper Orthogonal Values (POVs) [7][8]. Since we mostly interested in the POMs and POVs, these are more conveniently calculated by an eigenvalue decomposition of the symmetric matrix

\[
XX^\top = US^2U^\top.
\]

Typically the majority of the response is within a subspace of low dimension, and this is clearly shown by the POVs, which have high values for those POMs that contribute significantly to the response. Thus the POVs are ordered and a threshold decided, for example based on the percentage of energy retained in the response [8]. Thus, the POMs are split into two sets, given by

\[
U = \begin{bmatrix} U_r, U_e \end{bmatrix}
\]

where the subscript \( r \) denotes those POMs retained and the subscript \( e \) denotes those POMs neglected.

2.1. Quantifying subspace overlap

The retained POMs define the subspace of the significant response of the structure, and the question is then how closely two response subspaces overlap. Several measures to quantify the degree of overlap of two subspaces will now be given, where the subspaces are defined by the orthogonal basis matrices \( U_1 \) and \( U_2 \).

2.1.1. Angle between subspaces

The principle angle, \( \theta \), between the subspaces is calculated from the matrix

\[
C = U_1^\top U_2.
\]

Since the matrices are orthogonal, the singular values of \( C \) are less than 1, and hence the principal angles, \( \theta_i \), are defined by

\[
\cos \theta_i = \sigma_i
\]
where $\sigma_i$ is the $i$th singular value. The subspace angle $\theta$ is then

$$\theta = \max (\theta_i).$$

(7)

2.1.2. Residual projection error

The subspaces $U_1$ and $U_2$ were obtained from the sampled time response matrices $X_1$ and $X_2$ respectively. The choice of the subspace was based on capturing most of the corresponding response. Hot et al. [8] suggested an alternative measure, where the amount of the response $X_2$ captured by subspace $U_1$ is determined. This is based on the projection matrix $U_1 U_1^T$, and hence the response $X_2$ within the subspace defined by $U_1$ is

$$\hat{X}_2 = U_1 U_1^T X_2$$

(8)

and hence the residual is

$$r = \|\hat{X}_2 - X_2\| = \|U_1 U_1^T - I\| X_2\|.$$  

(9)

3. UNDERSTANDING THE SPATIAL RESPONSE OF A NON-LINEAR SYSTEM

The response of a linear systems is generally best understood using the mode shapes of the structure; the excitation applied to a structure is transformed into the excitation of individual modes, and the response is obtained as a summation of contributions of these modes. Usually the number of degrees of freedom is very high, and hence only a small subset of lower frequency modes are of interest. In non-linear systems there are difficulties with this approach, specifically the absence of a set of constant mode shape vectors, and the existence of multiple solutions for a given excitation. However, for weak non-linearities the response can be interpreted in the modal space of the linear system, with the non-linearities introducing interactions between a reduced set of the linear modes. In this way the non-linear response is reduced to the modal participation factors of the linear structure. Alternatively the modal transformation may be viewed as a projection of the non-linear response onto the subspace spanned by the mode shapes of the linear system. Using the POD is a similar approach, where the response is projected onto a different subspace based on the POMs. There are several issues with this approach that will now be discussed in more detail.

An important question for the reduced order model is the extent to which the dynamics of the full system is captured. In terms of the projection onto a subspace, the question is the amplitude of the residual of the dynamic response of the full system that is outside the subspace. Often the number of linear modes included in the reduction transformation is higher for a non-linear structure than a linear structure; this is a reasonable approach to allow for increased spatial complexity in the non-linear response. However, it should be emphasised that the linear modes may not be the optimum basis for the response, and it may be that the POMs are better able to capture the response. If the model reduction captures most of the dynamics, then clearly the modal interactions of interest will occur within the projected subspace.

A non-linear structure will often produce harmonics of the excitation frequencies, and hence will transfer energy from low frequencies to high frequencies. In linear systems high frequencies always means higher modes and a more complex spatial response. This is not necessarily the case in non-linear system; depending on the form of the non-linearity, the excitation and the particular solution obtained, the change in the spatial response may look similar to a range of linear modes.

One difficulty with the POD is that it requires the response to a particular excitation from a particular initial condition to obtain the POMs. This response should be representative of the response of the structure of interest during the analysis. For example, if the structure is able to exhibit multiple solutions, and only one solution is used to obtain the POMs, then there is no guarantee that the reduced model based on these POMs will correctly capture the other solutions.
4. NUMERICAL EXAMPLE: BOWSTRING ARCH BRIDGE

4.1. Description of the structure. Simulation

The Palma del Río Viaduct is located in Córdoba (Spain), over the Guadalquivir river. The viaduct is a steel composite bowstring arch opened to traffic in 2008, and it consist of a 130m long simply supported span (Figure 1). The bridge was defined as a steel-composite solution (Fig. 2), consisting on two inclined steel bowstring arches and a composite deck. The inclination of the arches is 68.8 degrees and it has a parabolic profile with a maximum rise of 25m. The elements of the steel arches are defined with tubular sections 900x50mm. The arches are tied by a Network pattern of cables of 45mm of diameter, made of steel Y1860. The concrete slab has geometrical dimension of 16m width and 0.25m thickness, connected to the arches by steel traversal braces. These braces has a variable cross section (Figure 3) defined by double T profiles, following a circular curve of 60m of radius for the lower flanges, and a linear variation in the rest. They have a maximum web of 1.25m in the center and they are located each 5m. The supporting piers are realized in a reinforced concrete solution and they are founded on piles.

Figure 1: Panoramic view of viaduct of Palma del Ro

Figure 2: Cross section

Figure 3: Traversal braces. Bridge under construction.
4.1.1. Finite Element Model. Time history simulation under moving loads.

The Finite Element Model of the structure was fully developed in order to count on a model as much accurate as possible. The bridge is modeled with 536 frames, 62 cable elements and 113 shell elements in SAP2000v15 as shown in Fig. 4. The shells are defined as thin shells with two sections, 0.25m and 0.02m thick for the concrete slab and steel plates in the top sections of the arches respectively, made of concrete HA-30 and steel S355. On the other hand, the braces between the arches are defined with tubular cross sections of 219.1x10mm and steel S355. In order to simulate the effect of the increased reinforcement in the concrete deck through its transverse direction, the beam rigidity of the concrete deck was increased by 20%, a good approximation in experience. The boundaries are the common in a bridge of simply supported spans, translational fixed bearings in one side and longitudinal fixed movement in the rest of supports.

The loading condition was defined with two moving loads, representing two vehicles crossing the structure in opposite directions. The loading lanes were defined in the center of the roads, 1.3m from the center (Figure 5). The loads were defined as two Kronecker deltas of 1t weight, running with 20m/s and 15m/s respectively. The time history analysis was carried out by Newmark-Beta method, with 500 steps and a time of integration of 0.02s. The damping was defined with a Rayleigh model, with damping ratios of 2% for the first two natural frequencies, 1.36 and 1.58 Hz respectively.

Figure 4: Finite Element Model

Figure 5: Moving load simulation scheme

4.2. Data acquisition

4.2.1. Measurement layout

Structural Health Monitoring (SHM) is the technology which led the maintenance politics into a more effective stage, through a more reliable knowledge of the actual behavior of the structure what provides more realistic computational models. It is possible to include some parameters so that to simulate the
existence of structural failures, hence, structural damage identification can be also carried out. Identification of dynamic characteristics of the system (mode shapes, natural frequencies and damping ratios) from the accelerations records, the classical techniques of Experimental Modal Analysis (EMA) obtains the dynamic properties under well-known forces. However, this technique is toughly scaled to civil structures because of their huge dimensions. On the contrary, in last years the appearance of Operational Modal Analysis (OMA) permits the monitoring of a structure with no need to know the exterior excitements. This technique permits the monitoring of the structure in service, with no need to install excitation devices on the structure. This technique was proposed by Farrar et al. [19], from the James et al. works [20]. The equipment used for the ambient vibration tests includes 14 uniaxial accelerometers, a 17-channel data acquisition system (Figure 7), and approximately 300m of uniaxial accelerometer cables (Figure 6). The minimum frequency range and sensitivity of the accelerometers are 0.1 Hz and 10V/g, respectively. Ambient vibration tests selected the frequency span of each measurement to be 0-50 Hz. Total duration of each measurement test setup was 30 min. The accelerometers were placed as shown in Figure 7. The span was divided in two lines, 7.8m from the center, in where the accelerometers were placed in four set-ups. Two references were kept steady along the whole process so that the set-ups could be related later to define the mode shapes. Only vertical responses (+Z) were measured. The collected signals were transferred into an own software.

In Figure 8 the numerical and experimental time series of accelerations for the first four sensors in set up 1 are represented. Fast Fourier Transformations of these signals were carried out in order to check the frequency range excited by forcements. In Figure 9 the first four FFTs of the first four sensors,
numerical and experimental, are represented for the set up 1. Note we came up against a spurious peak at frequencies next to origin, probably due to some disturbances introduced by the cables of the accelerometers. However, we obtained good correlation in the rest of the peaks corresponding to natural frequencies of the structure.

![Figure 8: Time series of four first sensors (numerical and experimental) in set up 1.](image1)

![Figure 9: FFT of numerical and experimental time series for first four sensors in set up 1.](image2)

### 4.3. Detection of non-linear structural behavior

According to the methodology defined by Ritto [6], the time response histories of the four set ups were assembled separately. Then, by diving the signals in 10 windows, the SVD transformation was carried out and the POMs were obtained. Only the contribution of the first three POMs were taken into consideration. Finally, the subspace overlaps between experimental and numerical responses were carried out by equation 5. In Figure 10 the angles between linear and non-linear subspaces are represented for the three considered POMs. The maximum angles were collected (Eq. 7) for the three POMs and the four set ups (Figure 11). The overlaps are very sensitive to the number of signals taken into consideration, increasing drastically with the number of sensors.
For the first set up, Figure 11 shows the overlap evolution when the fourteen sensors were chosen at the same time. The evolution of overlaps through every window of the response maybe observed. Thus, in the first set up windows 3, 4 and 5 give the maximum angles. In the rest of the set ups the maximum values were obtained in windows (8-9-10), (1-2-3) and (3-4-5) respectively. Angles are significantly greater than zero, so the results show some considerable non-linear behavior.

Figure 10: Angles between first three mode shapes and POMs. Set up 1, 4 sensors.

Figure 11: Maximum angles between linear and non-linear signals, 14 sensors.

5. CONCLUSIONS

Non-linear detection methodologies have been introduced. Relationships between the Proper Orthogonal modes and non-linear detection have been discussed. A field testing was carried out in the viaduct of
Palma del Río. Some disturbances in the measures were detected, probably because of noise introduced by the cables of the accelerometers. A 3D model of the structure was developed and similar exciting conditions were implemented by a Time History Analysis. The Effective Independence method was employed with Proper Orthogonal modes to quantify the non-linear behavior of the structure. The outcomes of these results show the potential of the proposed methodology and will be the subject of further investigation.

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