A NEW TECHNIQUE FOR DAMAGE LOCALISATION USING ESTIMATES IN KREIN SPACES

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ABSTRACT
In this paper a vibration based damage identification technique for structural health monitoring systems, based on identified state space models of randomly excited real world structures, is described. Thus a laborious experimental set-up – e.g. impulse response functions of large scale experiments of bridges, towers etc. – can be omitted. Applying the developed vibration based damage identification technique, simulation and laboratory tests were conducted at the Leipzig University of Applied Sciences. Here a steel rectangular hollow section – build as a cantilever arm (bending beam) – was excited by ambient vibration and the structural response was measured by eight uniaxial piezoelectric accelerometers. Build up on estimates in Krein spaces, a new vibration based identification method, which is capable of detecting and localising structural damage for structural health monitoring purposes, was applied. It must be emphasised that this method is based on the complete parametrisation of state space models \( S \{ A, B, C, D \} \), in contrary to classical Output-Only methods \( S \{ A, C \} \).

Keywords: Structural Health Monitoring, System Identification, Subspace Methods, Linear Algebra, Krein Space Estimation

1. INTRODUCTION
Modal testing of mechanical structures for the extraction of features like frequencies, dampings and modeshapes etc. are state-of-the-art in vibration based damage identification techniques for structural health monitoring systems. Over the last decades these techniques were extended by subspace identification methods, in which estimates in Hilbert spaces were used. Even though some of these methods like Output-Only methods are at this time successfully used to determine frequencies, dampings and modeshapes etc., the whole transfer behaviour with additionally computed amplitudes and phases of the analysed structure can be identified only if input forces are known. Damage identification of large scale experiments build on complete – parametrised with MIMO state space models – transfer functions and derived characteristics, such as markov parameters, is actually working as shown in [1,2,3,4] in which a
62.5m long tied-arch bridge in Hünxe (North Rhine-Westphalia, Germany) was analysed. In this experiments an induced damage could be localised. Here a transfer behaviour described by a state space model is promised to be much more significant than comparable models found from mechanical principles. Within the framework of previous studies, experiments – including deterministic signals for example impulse response functions – were conducted. This type of input signals (e.g. impulse or sweep functions) are difficult to apply to large structures, which are usually inspected in the area of civil engineering such as bridges, towers etc. To face this poor circumstances, theoretical and numerical challenging stochastic system identification methods – depending on ambient excitations like wind, vehicular traffic etc. – can be applied instead. Here in place of measuring input disturbances, which is not possible, statistical parameters are assumed a priori.

In this paper a vibration based damage identification technique for structural health monitoring systems, based on identified state space models of randomly excited real world structures, is described on the basis of theoretical considerations and laboratory tests. Here a structural damage could be detected and localised in laboratory experiments by employing physical interpretable indicators, which were build up on estimates in Krein spaces.

2. THEORETICAL BACKGROUND

Structural Health Monitoring is an interdisciplinary research field. Examples can be found in the area of civil engineering (e.g. bridges, towers), aerospace engineering (e.g. planes, helicopter), mechanical engineering (e.g. robots, rotor blades of wind energy plants) and so on. There are many applied examples, like the following monitored bridges in the area of civil engineering: Huey P. Long Bridge (Louisiana, USA), Rio-Andirrio Bridge (Greek), Tsing-Ma-, Ting-Kau-, Kap-Shui-Mun-Bridges (Hong Kong, China).

In preparation of the conception of monitoring systems, the design engineer has to choose which components should be supervised. These components can be monitored differently. Therefore quantities to be measured (e.g. strain, vibration velocity or acceleration) and measurement points must be chosen. By planning a monitoring system one can describe it by certain levels [5]:

1. **Detection** Is there damage in the system?
2. **Localisation** Where did damage appear?
3. **Assessment / Classification** What type of damage emerged in which scale?
4. **Prediction** How long is the systems remaining residual life?

2.1. Modelling Mechanical Structures

A central question in a model based monitoring concept is the kind of model which should be used for the mechanical structure and the damages occuring during its lifespan. Here three modelling approaches must be distinguished:

**White-Box-Model** The model is build up on analytical physical principles (e.g. in structural mechanics: static or dynamic equilibrium equations, geometric kinematic compatibility conditions, material law).

**Black-Box-Model** A system is parametrised as a causal transfer operator from system excitation (e.g. forces) to system response (e.g. vibration acceleration; s. fig. [1]).

**Grey-Box-Model** Both explained modelling types are used together in a hybrid form.
White-Box models are often developed with Finite Element Models, in which geometry, materials and boundary conditions are defined a priori. This type of models can be verified by comparing predicted and measured model behaviour. This process is called Model Updating. By applying this procedure one has to choose among a great amount of defined parameters a subset of those which shall be modified corresponding to the conducted experiments.

In contrary to White-Box models, which are based on physical principles, Black-Box models present a parametrised causal transfer behaviour from input to output. These causal models are build up on excitations, which can be measured or assumed a priori by statistical parameters, and measured response functions.

Black-Box models can be described in different ways. An advantageous method is the state space representation, because of its advantageous approach to MIMO systems. Central element of this kind of modelling is the not measurable state \( x \in \mathbb{C}^n \), which acts as an intermediate step between system input and output. The discrete stochastic linear time-invariant state space model is defined here as follows.

\[
\begin{align*}
    x_{k+1} &= Ax_k + w_k \\
    y_k &= Cx_k + v_k
\end{align*}
\]

Herein we use the system response \( y \in \mathbb{R}^p \), the process noise \( w \in \mathbb{R}^n \) and the inevitable measurement noise \( v \in \mathbb{R}^p \) as a result of electrical noise, numerical inaccuracies of AD-conversion and numerical methods, etc. The shown model can be expressed equivalently in the z-domain.

\[
y(z) = C(zI - A)^{-1}w(z) + v(z)
\]

When analysing spectra, the term \( z = e^{j\omega} \) is applied (herein \( \omega \) is the angular frequency).

### 2.2. System Identification

System identification methods for Black-Box models are methods in linear algebra, which are used to parametrise state space variables on the basis of known system excitations and responses (e.g. measured signals like forces, acceleration, velocity; s. above). For choosing an identification method one have to distinguish between systems induced by deterministic and stochastic excitations. One large-scale experiment, in which a bridge near by Hünxe (North Rhine-Westphalia, Germany) was identified deterministically, is described in [1, 2, 3, 4, 5]. This 62.5m long tied-arch bridge was identified as a Black-Box model with deterministic methods based on impulse response functions and – by comparing identified models – structural damages could be detected and localised. Deterministic identification methods – based on impulse response functions – use special inputs (dirac delta functions \( \delta(t) \)). As the authors showed, theoretical described impulse functions cannot be applied correctly to large-scale structures, which are usually found in civil engineering. Because most mechanical structures are induced permanently by ambient noise (e.g. bridges under wind loads and vehicular traffic), elaborate deterministic test loads can be substituted for stochastic ones. Therefore stochastic system identification methods shall be employed.

Stochastic system identification methods are based on measured responses and random excitation functions, which are assumed a priori by white noises with statistical parameters. At first a well known Output-Only-Identification is proceeded to get the dynamical parameters \( S \{ A, C \} \) of the investigated system, which is called the first identification step. By estimating the phases and amplitudes, the whole transfer behaviour of the investigated mechanical system can be modelled. In this so called second identification step the theory of the classical Kalman Filter is used to get estimated states \( \hat{x}_{k+m[k] \rightarrow x_k} \), which can
be geometrically interpreted as an orthogonal projection of a priori assumed states $x_k$ onto the data space $y_k$ (a linear combination of observations $y_0, y_1, \ldots, y_k$). The difference $\tilde{x}_{k+m|k} = x_{k+m} - \hat{x}_{k+m|k}$ is called the estimation error and is used as the performance index to be minimized. This is implemented through an estimation error, which is orthogonal to the data space (orthogonal projection).

$$\min E\left\{\|x_{k+m} - \hat{x}_{k+m|k}\|^2\right\} = \min E\left\{\tilde{x}_{k+m|k}^2\right\}$$

(4)

The estimation with Kalman filters reduces the expected estimation error energy, as we can see in the above equation. If we furthermore assume Gaussian noises, the Kalman filter becomes a Least-Mean-Squares (LMS) optimization.

By choosing $m$ one specifies the estimation problem as smoothing $m < 0$, filtering $m = 0$ and prediction $m > 0$. In the following we concentrate on One-Step-Predictions with $m = 1$ and for simplicity we write the one-step predicted states as $\hat{x}_k$.

The classical Kalman filter, which is a dynamical system with a feedback-loop, can be used to compute recursively the estimated states $\hat{x}_k$.

$$\hat{x}_{k+1} = (A - KC) \hat{x}_k + Ky_k$$

(5)

By reordering this equation, we now define the innovations process $e_k = y_k - C\hat{x}_k$, which is a causal process. Thus a replacement of the original model – the so called innovations form – can be developed (s. eq. 5).

$$\hat{x}_{k+1} = A\hat{x}_k + K\left(y_k - C\hat{x}_k\right)$$

$$y_k = C\hat{x}_k + e_k$$

(6)

(7)

For identification purposes the innovations form can be improved by introducing a target function $z_k = Lx_k$ and its estimation $\hat{z}_k = L\hat{x}_k$. In this context the linear combination matrix of the states is chosen as $L = C$, so a noise reduced output $\hat{y}_{k|k-1} = C\hat{x}_k$ follows. Therefore the full parametrised model for the mechanical system is defined by $S\{A, K, C, 0\}$. Thus the following task is to compute the optimal Gain $K$.

In the case of time invariant systems and stationary noise processes, the computation of $K$ follows the factorization of the spectral density $\Phi(z)$ of the data $y$. Here $W(z)$ is the so called spectral factor, which is found in the first identification step.

$$\Phi(z) = \sum_{l=-\infty}^{\infty} E\left\{y_{k+l}y_k\right\} z^{-l} = \sum_{l=-\infty}^{\infty} A_{yy}(l)z^{-l}$$

$$= W(z)W^*\left(z^{-*}\right) = \left[C\left(zI - A\right)^{-1} B + D\right]\left[D^T + B^*\left(z^{-*}I - A^*\right)^{-1} C^*\right]$$

$$= \left[C\left(zI - A\right)^{-1}\right]\left[Q \begin{bmatrix} S & S^* \end{bmatrix} \begin{bmatrix} I & (z^{-*}I - A^*)^{-1} C^* \end{bmatrix}\right]$$

$$= \left[C\left(zI - A\right)^{-1}\right]\left[Q \begin{bmatrix} S^* & R \end{bmatrix} \begin{bmatrix} K^* & \tilde{K} \end{bmatrix}\right]$$

$$= \left[C\left(zI - A\right)^{-1}\right]\left[Q \begin{bmatrix} S^* & R \end{bmatrix} \begin{bmatrix} K^* & \tilde{K} \end{bmatrix}\right]$$

$$\geq 0$$

(12)
To compute the factorization of the spectral density, we need to solve a single Riccati-Equation or a system of Riccati-Equations. There is a huge variety of possibilities of how to solve this operation. We present here the Discrete-Algebraic-Riccati-Equation (DARE)

$$P = E \{ \ddot{x} \ddot{x}^* \} = APA^* - (APC^* + S)(CPC^* + R)^{-1}(APC^* + S)^* + Q$$  \hspace{1cm} (13)

and presume the computation of a generalized Schur form of an extended symplectic pencil. In the framework of this paper we cannot go further in detail here (for details s. [8]).

After the determination of the error covariance matrix $P$, the Kalman Gain $K = (APC^* + S)R_e^{-1}$ and the covariance matrix of the innovations process $R_e = E \{ ee^T \} = CPC^* + R$ are ready to be computed.

A huge problem results from the estimation of the spectral factor $R$ of a solution can be found in [9].

showed in [10], that multivariate systems, which are typical in civil engineering, are hard to identify, the Riccati-Equation cannot be solved and the method fails to identify the system. Therefore the authors showed in [10], that multivariate systems, which are typical in civil engineering, are hard to identify, because of numerical problems. A further description of this problem and theories about the existence of a solution can be found in [9].

2.3. $H_{\infty}$-System-Identification

To avoid unfeasible estimation errors, the system excitations must be known perfectly in $H_2$-estimation. In the author’s laboratory experiments mechanical systems tend to have large estimation errors near by eigenfrequencies (frequency domain). To overcome this problem, $H_{\infty}$-estimation methods – which bound the estimation error – were introduced. This methods improved the damage identification procedures of the authors investigations and shall be shown in the following.

$H_{\infty}$-estimation methods can be explained easily with the help of Game Theory: Central objective is the defined target function $z_k = Lx_k$ and its estimation $\hat{z}_k = \hat{L}\hat{x}_k$. In this imaginary game, player 'nature' starts by worsening the estimation error $\hat{z} = z - \hat{z} = L\hat{x}$ to the maximum with the help of the noise processes $w$, $v$ and the initial state $x_0$. In Kalman filtering this player is indifferent, which is an optimistic view in contrast to the pessimistic modelling of the nature in $H_{\infty}$-estimations. After the first player maximized the error, the second one named 'designer' minimizing the estimation error $\hat{z}$ by the choice of the estimate $\hat{z}$. This so called minimax problem can be expressed adequately by the following performance index [11][12]. Here we use the two-norm (e.g. $\|w\|_{Q^{-1}}^2 = \sqrt{w^TQ^{-1}w}$).

$$\min_{\hat{z}_k} \max_{u_k, v_k, x_0} \frac{\sum_{k=0}^{N-1} \| z_k - \hat{z}_k \|^2}{\| x_0 - \hat{x}_0 \|^2 + \sum_{k=0}^{N-1} \left( \| w_k \|^2_{Q^{-1}} + \| v_k \|^2_{R^{-1}} \right)}$$  \hspace{1cm} (14)

There are a few analytic solutions to this minimax problem in special cases only. So we substitute the minimization with a user-defined bound $\gamma$. After further simplifications the criterion becomes:

$$\max_{u_k, v_k, x_0} \frac{\sum_{k=0}^{N-1} \| \hat{z}_k \|^2_{L^2} + \sum_{k=0}^{N-1} \left( \| w_k \|^2_{Q^{-1}} + \| v_k \|^2_{R^{-1}} \right)}{\| \hat{x}_0 \|^2_{P_0^{-1}}} < \gamma^2$$  \hspace{1cm} (15)

For time invariant systems and stationary noise processes, as presumed in this paper, this criterion is equivalent to the computation of the greatest singular value of a system $G(e^{j\omega})$, which maps the noise processes $w$ and $v$ (denominator) to the estimation error $\hat{z}$ (numerator). So we deal with the infinity-norm $\| \cdot \|_{\infty}$ by employing the two-norm (e.g. $\| x \|^2 = x^T x$).

$$\sup_{w, v \neq 0} \frac{\| \hat{z} \|^2_{L^2}}{\| w \|^2_{2} + \| v \|^2_{2}} = \sup_{\omega} \sigma_{\max} \left[ G(e^{j\omega}) \right] = \| G \|_{\infty} < \gamma^2$$  \hspace{1cm} (16)

Based on the defined performance criterion, an identified system shall be developed comparable to the $H_2$-case. Therefore we need to estimate states (s. eq. [6]) and define the estimated target function
\[ \dot{z} = \hat{y}_{k|k-1} = C \hat{x}_k \] with \( L = C \). So the whole transfer function \( S \{ A, K, C, 0 \} \) can be computed, after the dynamics are estimated by classical Output-Only methods. Thus the determination of the Gain \( K \) follows, which can be done by applying Lagrange Multiplier [12] or by computing a J-spectral factorization [13]. In the following the second approach shall be shown.

The J-spectral factorization is defined – based on the estimation error system in eq. [16] and the noise processes in eq. [12] - as follows [13] (comparable to eq. (9)).

\[ \Psi(z) = W(z)JW^*(z^{-*}) = \left[ \begin{bmatrix} C \\ L \end{bmatrix} (zI - A)^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix} + \begin{bmatrix} D \\ 0 \end{bmatrix} - \begin{bmatrix} I \\ 0 \end{bmatrix} \gamma^2I \right] \times \left[ \begin{bmatrix} B^* \\ 0 \end{bmatrix} (z^{-1}I - A^*)^{-1} \begin{bmatrix} C^* \\ L^* \end{bmatrix} + \begin{bmatrix} D^* \\ 0 \end{bmatrix} - \begin{bmatrix} I \\ 0 \end{bmatrix} \right] \]

(17)

(18)

This factorization can be optimized, which can be interpreted geometrically as projections in Krein spaces. Here projections in Hilbert spaces are not possible, because of the indefinite nature of \( R_e \).

\[ \Psi(z) = \left( \begin{bmatrix} C \\ L \end{bmatrix} (zI - A)^{-1} K_p + I \right) \langle \langle e e^T \rangle \rangle \left( K_p^* (z^{-1}I - A^*)^{-1} \begin{bmatrix} C^* \\ L^* \end{bmatrix} + I \right) \]

(19)

\[ R_e = \begin{bmatrix} R \\ 0 \\ 0 \\ -\gamma^2I \end{bmatrix} + \begin{bmatrix} C \\ L \end{bmatrix} P \begin{bmatrix} C^* \\ L^* \end{bmatrix} = \begin{bmatrix} R + CPC^* \\ LPC^* \end{bmatrix} \begin{bmatrix} C^* \\ L^* \end{bmatrix} + \gamma^2I + LPL^* = R_{e}^{1/2} \begin{bmatrix} I \\ 0 \\ 0 \\ -I \end{bmatrix} R_{e}^{-1/2} \]

(20)

Comparable to the classical Kalman filtering case, a Riccati-Equation or a system of Riccati-Equations must be solved to conduct the factorization. Here we show the Discrete-Algebraic-Riccati-Equation (DARE). Computational concerns are described in [8].

\[ P = APA^* - \left( AP \hat{C}^* + \hat{S} \right) \left( \hat{C}P \hat{C}^* + \hat{R} \right)^{-1} \left( AP \hat{C}^* + \hat{S} \right)^{+} + Q \]

(21)

\[ \hat{C} = \begin{bmatrix} C \\ L \end{bmatrix} \hat{R} = \begin{bmatrix} R \\ 0 \\ 0 \\ -\gamma^2I \end{bmatrix} \hat{S} = \begin{bmatrix} S \\ 0 \end{bmatrix} \]

(22)

This equation can be computed equivalently by the following.

\[ P = APA^* - \left( AP \hat{C}^* + S \right) \left( \hat{C}P \hat{C}^* + R \right)^{-1} \left( AP \hat{C}^* + S \right)^{+} + Q \]

\[ \hat{P} = P \left( I - \gamma^{-2}L^*LP \right)^{-1} \]

(23)

(24)

The solution \( \hat{P} \) can now be used to create a \( H_\infty \)-One-Step-Predictor [13].

\[ K = (AP \hat{C}^* + S) \left( R + CPC^* \right)^{-1} \]

(25)

For identification purposes we use \( L = C \) and the parametrised system \( S \{ A, K, C, 0 \} \) follows.

In the last statements in this section we realize the connection between \( H_\infty \) and classical Kalman-predictors. In Kalman theory the linear combination matrix of the states \( L \) is not important for the solution, in contrast to the \( H_\infty \)-case. Furthermore a user-defined bound \( \gamma \) is applied, which results in an increased error matrix \( \hat{P} > P \) (s. eq. [24]), which can be described as being cautious against the defined model and noise presumptions. For this reason a Kalman filter is reconstructed with the choice of a bound \( \gamma \rightarrow \infty \).
2.4. Damage Indicator

Here a new defined damage indicator for vibration based damage identification methods on the basis of Krein space estimations is demonstrated. The fundamental idea behind this new damage vector $d$ is to subtract the outputs of two systems. If the compared systems have identical or similar properties and are excited adequately, the resulting difference should be approximately zero. As soon as one system changes its properties, the damage vector alters. Therefore a monitored mechanical structure is identified as a Black-Box model at different times:

1. Reference state $H_{id,1}$ after starting the monitoring system (e.g. after completion of the building).
2. Arbitrary state $H_{id,2}$ after the first inspection in consequence of degradation, etc. of the structure.

In the following the monitoring system shall detect and localise structural damages. So the identified systems $H_{id,1}$ and $H_{id,2}$ are induced by a causal noise and their responses are subtracted.

$$d(z) = H_{id,2}(z)e_2(z) - H_{id,1}(z)N(z)e_2(z)$$

(26)

Both transfer behaviours must be scaled to the same excitation to be comparable. Therefore the transfer matrix $N(z)$ is introduced. In the following $D = \langle dd^T \rangle$ is analysed. Here differences from the expected value indicates the location of structural damages (2nd level of damage identification).

3. EXPERIMENTAL VERIFICATION

Applying the developed vibration based damage identification technique, laboratory tests were conducted at the Leipzig University of Applied Sciences. Here a steel rectangular hollow section – build as a cantilever arm (bending beam; $E = 2.1 \cdot 10^{11} N/m^2$, $I = 17.7 \cdot 10^{-8} m^4$, $l = 2.45 m$) – was excited by ambient noise and the structural response was measured by eight equally spaced (30 cm), uniaxial, piezoelectric accelerometers ($\Delta t = 10^{-3} s$). The mentioned set-up is shown in figure 2.

![Set-up used for the experiments](image)

Figure 2: Set-up used for the experiments

After the measurement of vibration accelerations, the signals were filtered and digitised. Then the extracted stochastic signals of 1 hour length were used to estimate an averaged power spectral density (average of ≈ 220 estimates). The power spectral density estimate and its inverse Fast Fourier Transform – covariance matrices – were the basis of the following system identification.

In the first identification step the dynamic of the investigated system (e.g. eigenfrequencies, dampings, unscaled mode shapes) was analysed by the use of covariance based Output-Only methods. As a result $S \{ A, C \}$ was estimated. Afterwards projections in indefinite Krein spaces were computed – in the second identification step – to parametrise the whole searched state space model. In the following this
identified state space model $S \{ A, K, C, 0 \}$ allowed the study of a causal transfer behaviour from system input to output. An example of the estimation at the end of the bending beam ($S_1$) is shown in fig. 3.

In this example the vertical bending eigenfrequencies $f_{e,2} = 34, 9Hz$, $f_{e,3} = 96, 4Hz$, $f_{e,4} = 184, 4Hz$, $f_{e,5} = 288, 5Hz$ and $f_{e,6} = 388, 7Hz$ are identified well, as we can see its peaks and positions fit to the measured data. Furthermore we recognise the reduction of noise terms between eigenfrequencies. The first vertical bending eigenfrequency was not identified here, because of the small excitement in the low frequency range.\(^1\)

Identifying and comparing state space models of reference and damaged states, structural damages were identified. Henceforth the study of the new defined physical interpretable damage vector was used for the detection and localisation of structural damage. The applied structural damage – or better called structural change – was applied by magnets ($m_{dmg} \approx 1kg$), so the experiments could be repeated for verification purposes.

In the damage identification process the damage indicator $D$ was analysed. Following the presented theory, a recognised deviation from zero indicates structural changes. Here the estimated probability density function was used to observe at which measuring location the greatest error transmission occured. One studied example is shown in fig. 4.

Using the new damage identification technique, the structural change at measuring position 5 (middle of the beam) could be detected and localised. Yet another example is shown in fig. 5. Here the structural change at measurement location 1 (end of the beam) could be identified without doubt too.

\(^1\)This excitement resulted from the construction of the used loudspeaker, which simulated ambient noise.
For verification purposes the damage identification technique was applied to measurements of the same state (e.g. the reference state). One result is shown in fig. [5]. Here a presumed threshold value of 50% is not exceeded and the absence of a structural change is identified correctly.

On the basis of the shown laboratory tests – the identification of structural changes of a cantilever arm (bending beam) – we illustrated the usefulness of the presented theories for damage identification purposes and the reduction of noise in measurements.

4. SUMMARY

In this paper a new vibration based damage identification technique on basis of Krein space estimations is presented. Central detail in this methodology is the modelling approach of the mechanical structure. Here mechanical systems are not build up as FE-models (White-Box models), but with the help of Black-Box models. These models reduce significantly the estimation error due to a priori model presumptions, which is advantageous for system identification intentions. Here the identification of Black-Box models is based on the numerical parametrisation of state space descriptions, which allows to analyse the causal transfer behaviour from system input to output. Comparing identified systems with the help of a new defined damage indicator, structural changes could be detected and localised in laboratory tests at the Leipzig University of Applied Sciences. In these laboratory experiments (cantilever arm – bending beam) we could show the significant reliability for noise reduction and damage identification purposes of the presented theories. Furthermore blind tests were conducted successfully to verify the experiments. Within the shown theories many fields need further research. One long-term research object of the authors is the employment and verification of an online Structural Health Monitoring concept at large-scale experiments (e.g. bridges, towers).
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