STABILIZATION DIAGRAMS USING OPERATIONAL MODAL ANALYSIS AND SLIDING FILTERS

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ABSTRACT
This paper presents a filtering technique for doing effective operational modal analysis. The result of the filtering method is construction of stabilization diagram that clearly separates physical poles from spurious noise poles needed for unbiased fitting. A band pass filter is moved slowly over the entire frequency spectrum of the measured data, and poles in the band are identified for each new filter position. In this way all poles are identified many times, making the physical poles stand out to make them immediately identifiable. The technique is illustrated using the Time Domain Poly Reference(TDPR) system identification algorithm on simulated data.

Keywords: Operational, Modal, Analysis, Stability Diagram, sliding filter

1. INTRODUCTION
When performing modal analysis the challenge comes down to extracting the most and best information from the experimentally measured data. In that process several parameters such as measurement equipment, excitation, identification technique, noise, preprocessing of the data, etc. has influence on the quality of the obtained information. When performing operational modal analysis(OMA), we are exploiting the ambient conditions as a random excitation to obtain a time signal from which we can extract the modal parameters such as the natural frequency, damping ratio and mode shape. In this case the correlation or spectral density function is extracted from the time signal and used to estimate the modal parameters. They will contain all information from the signal such as physical properties of the structure, properties of the loading filter, harmonic components and noise. This complicates the process when searching for the physical properties of a structure [1].

During decades stabilization or consistency diagrams has been a key tool in the sorting and selection of the estimated poles. The most widely used approach has been a two dimensional plot with the frequency
on the horizontal axis and the model order on the vertical axis. Often the plot is constructed with either
the frequency response function (FRF), the power spectral density function (PSD), the singular values of
the PSD or another modal indicator function in the background [2]. The purpose of building the stabil-
ization diagram is to overcome the challenge of choosing the correct model order when using parametric
identification techniques. If the model order is chosen higher then the number of modes in the signal it
will result in estimating the number of poles corresponding to the chosen model order containing both
physical and spurious poles. To overcome this limitation the stabilization diagram is build by systemati-
cally increasing the model order and combining all the solutions in the stabilization diagram. Where the
physical poles stand out from the spurious poles as vertical lines in the diagram as they will be situated
at the same frequencies regardless of the model order. The stabilization diagram is often expanded to
contain a ranking of the estimated poles by different levels of importance ranging from the consistency
of the poles, considering also the consistency of the damping and finally considering the mode shape
vector where each level is indicated with different symbols in the stabilization diagram. Recent years the
importance ranking has been improved and hereby created a more clear stabilization diagram by the use
of normalization and parameter constraint in the identification method [3] [4].

The stabilization diagram is not limited to having the model order on the ordinate axis but can be used in
a more general form where the horizontal axis is a modal quantity and the vertical axis is a variation of
the identification process. When using parametric identification techniques the question of model order
or oversizing the model is important in order to deal with noise modes and bias in the signal. Therefore
the model need to estimate more poles then modes present in the signal, leaving poles to fit the noise
modes[1]. One approach is to raise the model order another is to decrease the number of modes in the
signal by the use of band pass filtering and using a parametric identification technique with a fixed order.

In this paper a different approach to build a stabilization diagram then the classic is proposed. A low
order parametric model is used for the identification process, where a band pass filter is used to obtain
a oversizing of the model. The band pass filter is slid through the frequency range by incrementally
increasing the center frequency of the band pass filter and for each increment estimating the poles of the
band pass filtered signal. Hereby building a stabilization diagram with the center frequency of the band
pass filter on the vertical axis and one of the estimated modal parameters on the horizontal axis. This is
illustrated in on a simulated signal.

2. THEORY/METHOD

The general idea of the sliding filter stabilization diagram is to use a band pass filter and let this slide
through the frequency range from DC until Nyquist frequency $f_{\nu}$. For each step changing the center
frequency $f_c$ of the band pass filter by a frequency increment $\Delta f_c$ and estimating the modal parameters
by the use of a parametric identification technique, as illustrated in figure[1].

**Step 1:** Acquire and pre-process the time signal

**Step 2:** Band pass filter the signal

**Step 3:** Estimate the poles by a low order parametric ID. technique

**Step 4:** Add the estimated poles to the stabilization diagram

Repeat step 2 to step 4 by increasing the center frequency of the band pass filter by a frequency increment $\Delta f_c$, moving
the filter through the whole frequency range.

**Figure 1:** Principles of sliding filter stabilization diagram.
This leads to the possibility of building stabilization diagram on the basis of the modal parameters as e.g. plotting the center frequencies of the band pass filter on the vertical-axis and the estimated frequencies on the horizontal-axis, being a \( f_c/f_{est}\)-diagram as illustrated in figure 2. Another possibility could be using the center frequency of the band pass filter and the estimated damping ratio, being a \( f_c/\zeta_{est}\)-diagram, this could be done by using the modal parameters either with \( f_c \) or in combination with a corresponding modal parameter.

![Figure 2: Illustration of the principles of the sliding filter stabilization diagram.](image)

A keystone in this algorithm is the band pass filter. When performing OMA, in most cases the signal is acquired as a final time series and afterwards processed, so there is no need for real time processing. This opens for filtering the signal by use of FFT filtering, which avoids the issue of phase distortion. The FFT-filter is based on transformation of the measured signal to frequency domain \( Y(f) \) and multiplying by a window \( W(f) \) in the frequency domain. In this paper a Hanning window with 50 percent overlap has been applied, which is defined as equation (1).

\[
Y_c(f) = \begin{cases} 
Y(f) W(f) & \text{if } f_c - \frac{B_1}{2} \leq f \leq f_c + \frac{B_1}{2} \\
0 & \text{otherwise}
\end{cases}
\]

(1)

The band pass filter is defined by the center frequency \( f_c \), a width of the filter \( B_1 \) and a transition band \( B_2 \), as seen in figure 3.

![Figure 3: Illustration of the parameters used in the band pass filter.](image)
When using a band pass filter to reduce the information in the signal to a frequency band some considerations regarding the choice of the filter settings should be considered. The optimal settings would be to isolate one mode at a time, but as the modes have different energy level and damping ratio it will results in different settings of the filter, where the width of the filter $B_1$ is given by the half power bandwidth which is given by the damping ratio $\zeta$ and the natural frequency $f_n$ of a given mode, see equation (2).

\[ B_1 > \text{max} \left(2 \zeta f_n\right) \]  

(2)

Therefore a filter with a wide band is necessary to isolate the whole bell of the high frequency modes, whereas the same width would include several modes for the low frequency modes. This can result in the estimation of the low frequency modes is poor, and in some cases where one or several modes have significant higher energy level than other modes in the frequency band, the low energy modes are not estimated in the identification process. Whereas the opposite having a narrow band corresponding to the low frequency modes with a small damping ratio, the modes with low energy level and a narrow frequency bell, will be well estimated, but the higher frequency modes will not include sufficient of the frequency bell in the frequency band, which often lead to poor estimation of the modal parameters.

The second step in the approach is to estimate the modal parameters in each increment of the center frequency by the use of a parametric identification technique. In this paper a time domain techniques with a fixed low order has been used to estimate the modal parameters. When using time domain techniques in OMA the correlation functions are used as input, because they have the same proprieties as free decays [5]. In this paper the time domain polyreference(TDPR) identification technique is applied using two Autoregressive constants in the companion matrix [6]. An order of two in the TDPR results in the number of estimated poles will be twice the number channels and they will be in complex conjugated pairs hereby resulting in the number of estimated poles equals the number of channels. Applying this to the band pass filtered signal leaves poles to fit the noise modes resulting in better estimates of the modal properties.

In the process of building the stabilization diagram there are three important factors: filtering, identification and the increment of the center frequency $\Delta f$. The bandwidth has been discussed in previous paragraph, the identification technique and the model order effects the number of estimated poles for each step in the process. The center frequency increment effects the number of repetitions of the estimated poles, meaning setting a very low increment results in estimating the same pole many times, whereas setting a large increment results in few repetitions of each estimated mode, but also a reduction in the computational effort. The number of estimations of a pole $n_{est}$ is given by the increment of the center frequency and the band width of the band pass filter as shown in equation (3).

\[ n_{est} = \frac{B_1}{\Delta f_c} \]  

(3)

3. NUMERICAL TESTCASE

The stabilization diagram is illustrated on a time signal from a simulated system with 8 channels and 8 poles as shown in figure 4. The system is simulated as described in [1]. A sampling frequency of 400 Hz and a duration of the signal of 100 seconds was used. The 8 mode shapes was generated randomly and normalized. The frequencies and damping ratios of the modes used in the simulation are shown in table 1. A FFT based simulation method was used, the implementation is described in the fffsim function from the OMAtoolbox accompanying [1]. The signal was added artificial noise, by adding Gaussian distributed white noise to the signal.
Table 1: Modal parameters for the numerical test case.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$ [Hz]</td>
<td>20</td>
<td>60</td>
<td>65</td>
<td>90</td>
<td>90.1</td>
<td>145</td>
<td>150</td>
<td>180</td>
</tr>
<tr>
<td>$\zeta_n$ [%]</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>1</td>
<td>0.6</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Settings of the band pass filter

<table>
<thead>
<tr>
<th>Filter parameter</th>
<th>Wide band</th>
<th>Narrow band</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$ [Hz]</td>
<td>27</td>
<td>11</td>
</tr>
<tr>
<td>$B_2$ [Hz]</td>
<td>13.5</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta f_c$ [Hz]</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Figure 4:** The singular values of the power spectral density functions for the simulated 8 Degree of freedom system

The stabilization diagram is generated from the principles in figure 1 using the sliding band pass filter illustrated in figure 3 and the values given in table 2 using the settings for a wide band. In this case the identification technique TDPR has been used with an order of two. This results in 8 modes are estimated for each increment of the band pass filter.

**Figure 5:** Stabilization diagram build by using the estimated frequencies versus the center frequencies of the sliding band pass filter with the wide band settings of 2. The red lines indicates the frequencies of the simulated system.

The stabilization diagram is illustrated in figure 5a where all the estimated modes are plotted against each center frequency of the sliding band pass filter. Around each mode of the simulated system the estimated frequencies form a vertical line as seen in figure 5b whereas in between the modes the estimated frequencies form horizontal or slightly inclined lines.
Figure 6: Stabilization diagram build by using the estimated frequencies versus estimated damping ratios. The red lines indicates the frequencies of the simulated system.

Another illustration of the estimated modal parameters is shown in figure 6a where the estimated frequencies is plotted against the estimated damping ratios. Narrowing in on one mode as seen in figure 6b it is seen that the at each of the modes from the system the poles are approaching from a higher level of damping towards the actual value and concentrating about this.

Figure 7: Stabilization diagram build by using the estimated frequencies versus the center frequencies of the sliding band pass filter with the narrow band settings of 2. The red lines indicates the frequencies of the simulated system.

In figure 7 the stabilization diagram is build by the use of the values given in table 2 using the settings for a narrow band. Building the stabilization diagram by the use of a sliding band pass filter, the settings of the filter can results in some mode are not indicated in the stabilization diagram. This is seen by figure 5b compared to figure 7b. Where it is clear that mode no. 7 at 160 Hz is not indicated as a vertical line.
4. CONCLUSION

Historically stabilization diagrams has been build by increasing the order of a parametric identification technique and plotting this against the estimated poles. In this paper an alternative approach is presented. Where a sliding band pass filter and a low order parametric identification technique is applied to build the stabilization diagram. This approach shows clearly where the physical poles of the system are located by vertical lines, when plotting the estimated frequencies on the horizontal axis and the center frequencies of the band pass filter on the vertical axis, but also when plotting the estimated frequencies against the estimated damping ratios. This approach for building a stabilization diagram shows potential in locating the physical properties of a system, but the settings of the band pass filter should be chosen with care for the width of the band pass filter. For future works the possibility of using the stabilization diagram in the scheme of an automated identification process should be investigated. Also expanding the stabilization diagram to include symbols that indicate the importance level of each pole should be incorporated, raising the information level of the stabilization diagram and making it more user friendly.

REFERENCES


