Vibration serviceability of the stress ribbon footbridge with external tendons and bearings

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ABSTRACT: A prestressed concrete stress ribbon footbridge with external tendons and bearings was constructed in Hakusan city. Impact tests and damping free vibration tests were carried out to grasp the vibration characteristics. The results of the vibration tests show that several vibration modes existed in the critical ranges of natural frequencies for vertical and lateral vibration. In order to estimate the vibration serviceability performance for those vibration modes subjected to pedestrians walking, walking tests were carried out when two or seven people walked while listening to the metronome that corresponded to the resonant frequencies. Also, dynamic response analysis was carried out using the Hivoss guidelines.

1 INTRODUCTION

A prestressed concrete stress ribbon footbridge is a type of suspension bridge without towers. A conventional bridge has two kinds of internal tendons composed of the internal prestressing tendons and suspension tendons (primary tendons) that carry the dead load in the concrete deck. The new type of bridge (Morinowakuwaku Bridge, Photo 1) (Machi et al. 2002) has external tendons that are placed outside of the deck and function as prestressing tendons. It was constructed for the first time in the world in 2001. Moreover, the object bridge (Ohmaki-dondo Bridge, Photo 2) with the bearings and the external tendons (primary and secondary tendons) was constructed in Hakusan city in 2009.

The prestressed concrete stress ribbon footbridge is more susceptible to vibration due to pedestrians walking because of its lightweight structure and suspension footbridge. The designer had taken into account the vibration serviceability performance of the footbridge due to pedestrians walking.
As a result of the design process of the object bridge, the frequencies of the vertical and lateral vibration were expected to be in the critical ranges as follows:

- $1.25 \text{ Hz} \leq f \leq 2.3 \text{ Hz}$ for the vertical vibrations
- $0.5 \text{ Hz} \leq f \leq 1.2 \text{ Hz}$ for the lateral vibrations

Therefore this study uses the vibration test to grasp the dynamic behaviour of the bridge and to estimate the vibration serviceability performance using the guidelines (Hivoss guidelines 2007).

2 OBJECT BRIDGE

A side view, a detail of the abutment and a cross section of the object footbridge are shown in Fig. 1. The span length of the concrete deck is 83 m, the maximum gradient is 5%, and the sag is $L/80$ ($L$: span length). The features of the object bridge are as follows:

(a) External tendons
The tension force of the prestressing tendons increases as the span length of the live load acting on the deck increases. As a result, the external tendons were placed outside of the concrete deck in Morinowakuwaku Bridge, as shown in Fig. 2. In the object bridge, the tendons are composed of the internal prestressing tendons, the primary external tendons, and the secondary external tendons, as shown in Fig. 1(b),(c). The internal prestressing tendons are fixed at the ends of the
deck. The primary external tendons are placed under the deck because of the reduction of the dead load (Ogawa et al. 2006). These tendons are used in construction to carry the dead load. The secondary external tendons support the deck with the dead and live load using the struts.

(b) Supported by the rubber bearing
Though a conventional type of prestressed concrete stress ribbon footbridge is fixed at the abutments, the deck of the object bridge is separated from the abutments and supported by the rubber bearing (Ogawa et al. 2006). This result is as follows:
@ The concrete deck is influenced by the excessive tension force produced by the creep, drying shrinkage, live load, and temperature in the design of the sag (L/80).
@ The prestress introduced on the deck reduces less than 50% due to the design of the sag (L/80).

3 EXAMINATIONS

3.1 Measured points
The examinations were carried out in order to check the dynamic characteristics and to estimate the vibration serviceability performance of the footbridge. The measured points of the object footbridges are shown in Fig. 3. The measured sensor is velocity. The velocity was measured every 1/8 of the span.

![Figure 3: Measured points.](image)

3.2 Impact tests due to human weight
Impact tests were carried out in order to grasp the dominant frequencies of the footbridges. The impact tests in vertical directions were performed by two people who jumped from the chair (40cm height) at the same time. After loading the impact due to human jumping, the damping free vibration of the velocity response was recorded. The loading points were at each 1/8 of span of the centre and the edge of the effective width.

3.3 Damping free vibration tests
The damping free vibrations tests were carried out by two people who jumped continuously while listening to the metronome that corresponded to the resonance frequencies. After confirming that the resonance was performed, they stopped the exercise and identified the damping constant of each vibration mode. The damping constants were analyzed using the ERA method (Juang et al. 1985).

3.4 Walking tests
Since the object footbridge is a lightweight suspension footbridge, the frequencies of these bridges exist in critical ranges (walking range of the human step frequency at 2Hz). In order to investigate the vibration serviceability, the walking tests were carried out when two or seven people walked while listening to the metronome that corresponded to the resonant frequencies.

4 ANALYTICAL MODEL
A side view of the analytical bridge model is shown in Fig. 4(a). Also, a detail of the cross sec-
tion of that model is shown in Fig. 4(b). The stress ribbon deck, strut member, and lateral member are modelled as beam elements. The rubber bearings at the abutments are modelled as spring elements. The external tendons (primary and secondary tendons) are modelled as axial force elements.

In the real bridge, the primary tendons are connected to the deck through the saddle members, as shown in Photo 3. Here, the saddle member plays the role of a hinge and a slide in the axial direction on the design. In the analytical bridge model, the tendons are rigidly connected to the deck through the saddle members and rigid lateral deck members. It is constructed this way in order to consider the friction of the real bridge.

5 NATURAL FREQUENCY

The natural frequencies obtained from the impact and the damping free vibration tests in the winter (5 degrees) and the summer (32 degrees) seasons are shown in Table 1. Also the vibration modes corresponding to the natural frequencies obtained from the eigen-value analysis are shown in Fig 5.

According to the table, the natural frequencies obtained from the examinations hardly change with temperature. In addition, the results of the analytical frequencies are close to the results of the examinations. Although a conventional type of stress ribbon footbridge has a bending asymmetry 1st mode as the first vertical vibration mode, in this bridge, the first vertical vibration mode appeared as bending symmetry 1st.

The critical ranges for the natural frequencies of the footbridges subjected to pedestrian walking by the Hivoss guidelines are as the follows:

- $1.25 \, \text{Hz} \leq f \leq 2.3 \, \text{Hz}$ (for the vertical vibrations)
- $0.5 \, \text{Hz} \leq f \leq 1.2 \, \text{Hz}$ (for the lateral vibrations)

Two vertical vibration modes (Bending symmetry 2nd and Bending asymmetry 2nd) and a lateral vibration mode (lateral-torsion 1st) existed in the critical ranges.

6 DAMPING CHARACTERISTICS

The damping constants were identified by the ERA method using data obtained from the damping free vibration tests, as shown in Table 1. Each member’s equivalent damping constants were evaluated by the GA (Genetic Algorithm) method using the experimental damping constants.
and the ratio of the strain energy. The results are as follows: stress ribbon deck 0.018; external tendon 0.007; struts and lateral member 0.04; and rubber bearing 0.093. Also the analytical strain energy proportional damping constants were calculated from the results of eigen-value analysis. Each member’s equivalent damping constants are shown in Table 1. The results of the analytical strain energy proportional damping constants are close to the results of the experimental damping constants.

Fig. 6 shows the ratio of the strain energy in each member (each member’s strain energy / total strain energy) for each vibration mode. The mode number in Fig. 6 shows the vibration mode corresponding to Table 1 or Fig. 5. The vertical vibration modes (No.1, 2, 4, 6, 7, and 9) have large ratios of strain energy in the external tendons member (primary and secondary tendons). However, the lateral vibration modes (No.3, 5, and 8) have large and small ratios of strain energy in the deck and in the external tendons member (primary or secondary tendons). This illustrates why the damping constants of the lateral-torsion mode are larger than those of the bending mode.

Table 1 : Analytical and experimental frequencies.

<table>
<thead>
<tr>
<th>No</th>
<th>Mode shape</th>
<th>Frequency (Hz)</th>
<th>Damping constant</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Examination</td>
<td>Analysis</td>
<td>Examination</td>
<td>Analysis</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Winter</td>
<td>Summer</td>
<td>Winter</td>
<td>Summer</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Bending symmetry 1st</td>
<td>0.62</td>
<td>0.63</td>
<td>0.68</td>
<td>0.009</td>
<td>0.011</td>
</tr>
<tr>
<td>2</td>
<td>Bending asymmetry 1st</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
<td>0.010</td>
<td>0.009</td>
</tr>
<tr>
<td>3</td>
<td>Lateral-torsion 1st</td>
<td>0.93</td>
<td>0.93</td>
<td>0.98</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td>4</td>
<td>Bending symmetry 2nd</td>
<td>1.44</td>
<td>1.43</td>
<td>1.41</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>5</td>
<td>Lateral-torsion 2nd</td>
<td>1.86</td>
<td>1.85</td>
<td>1.91</td>
<td>0.016</td>
<td>0.007</td>
</tr>
<tr>
<td>6</td>
<td>Bending asymmetry 2nd</td>
<td>2.06</td>
<td>2.04</td>
<td>1.99</td>
<td>0.011</td>
<td>0.012</td>
</tr>
<tr>
<td>7</td>
<td>Bending symmetry 3rd</td>
<td>2.84</td>
<td>2.79</td>
<td>2.70</td>
<td>0.011</td>
<td>0.012</td>
</tr>
<tr>
<td>8</td>
<td>Lateral-torsion 3rd</td>
<td>3.20</td>
<td>3.20</td>
<td>3.04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>Bending asymmetry 3rd</td>
<td>3.69</td>
<td>3.64</td>
<td>3.47</td>
<td>0.009</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Figure 6 : Ratio of the strain energy.

7 VIBRATION SERVICEABILITY DUE TO WALKING

Since it was confirmed that two vertical vibrations (Bending symmetry 2nd and Bending asymmetry 2nd), and one lateral vibration (lateral-torsion 1st) existed in the critical ranges, this study estimated the vibration serviceability performance for each mode in the examination and the analysis.

7.1 Estimation from the walking tests

The walking tests were carried out for vertical or lateral vibration when two or seven people walked while listening to the metronome that corresponded to the resonant frequencies. The vibration serviceability was estimated using the acceleration transformed from the measured velocity data.

When two people walked at the frequencies for the vertical vibrations (Bending symmetry 2nd and Bending asymmetry 2nd), the maximum accelerations of the girder were 0.13 and 0.26
m/sec^2, respectively. Compared to the limit value of the international standards (BS5400 (Blanchard et al. 1977); 0.5/0.5 m/sec^2, ONT83 (Ontario government. 2000); 0.25/0.78 m/sec^2) or relevant literatures (Wheeler (Wheeler 1982) and Kajikawa (Kobori et al. 1974); 2.4 cm/sec), the maximum responses for vertical vibrations were not over the limit value.

On the other hand, when seven people walked at the frequency for the lateral vibration (lateral-torsion 1st), the maximum lateral velocities of the girder were 0.047 m/sec^2. Compared to the limit value of the international standards (Eurocode 1 (fib Bulletin 32. 2005); 0.14/0.5 m/sec^2), the maximum response for lateral vibration was not over the limit value.

7.2 Estimation from the analysis using Hivoss guidelines

Although the walking tests due to a few people could be carried out, the walking tests due to the crowded pedestrians could not be carried out. Then, the analytical vibration serviceability performance was estimated using the Hivoss guidelines.

Pedestrian traffic classes and corresponding pedestrian stream densities are proposed by the Hivoss guidelines. This study calculated the acceleration of the bridge in each design situation of the traffic class, as shown in Table 2.

<table>
<thead>
<tr>
<th>Traffic class</th>
<th>Density (Pedestrian/m²)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC1</td>
<td>1.5 P/(B L)</td>
<td>Very weak</td>
</tr>
<tr>
<td>TC2</td>
<td>0.20</td>
<td>Weak</td>
</tr>
<tr>
<td>TC3</td>
<td>0.50</td>
<td>Dense</td>
</tr>
<tr>
<td>TC4</td>
<td>1.00</td>
<td>Very dense</td>
</tr>
<tr>
<td>TC5</td>
<td>1.50</td>
<td>Exceptionally dense</td>
</tr>
</tbody>
</table>

The harmonic load models are provided for each traffic class TC1 to TC5. Depending on density, there are two different load models to calculate the response of the footbridge due to pedestrian streams:

- Load model for TC1 to TC3 (density d < 1.0 Pedestrian/m²)
- Load model for TC4 and TC5 (density d ≥ 1.0 Pedestrian/m²)

Both load models share a uniformly distributed harmonic load $p(t)$ [N/m²] that represents the equivalent pedestrian stream for further calculations:

$$p(t) = P \cos(2\pi f_s t) n' \psi$$

where $P \cos(2\pi f_s t)$ is the harmonic load due to a single pedestrian.

$P$ is the component of the force due to a single pedestrian walking at the step frequency $f_s$.

Also, $P$ is for vertical: 280 [N]; for longitudinal: 140 [N]; for lateral: 35 [N].

$f_s$ is the step frequency, which is assumed to equal the natural frequency of the footbridge under consideration.

$n'$ is the equivalent number of pedestrians on the loaded surface $S$.

$S$ is the area of the loaded surface.

$\psi$ is the reduction coefficient taking into account the probability that the step frequency approaches the critical range of natural frequencies under consideration.

The amplitude of the single pedestrian load $P$, equivalent number of pedestrians $n'$ (95th percentile), and reduction coefficient $\psi$ are defined (see Fig 7) considering the excitation in the first harmonic of the pedestrian load.

$n' = (10.8 \sqrt{\xi}/S)$ for TC1 to TC3 (density $d < 1.0$ Pedestrian/m²)

$n' = (1.85 \sqrt{n}/S)$ for TC4 and TC5 (density $d \geq 1.0$ Pedestrian/m²)

where $\xi$ is the structural damping ratio and $n$ is the number of the pedestrians on the loaded surface $S (n = S d)$. 
Moreover, the harmonic load $p(t)$ is applied to the footbridge for a particular mode shape, as shown in Fig 8.

This study calculated the maximum acceleration of the footbridge using the modal analysis ($\tau=0.01$ sec, $\beta=1/4$), which considers the harmonic load. Also, the strain energy proportional damping was assumed in this analysis, as shown in Table 1.

Fig 9 shows the results of the analytical maximum acceleration of the footbridge for vertical vibration (Bending asymmetry 2nd) and for lateral vibration (Lateral-torsion 1st), respectively. The comfort classes recommended by the guidelines in Table 3 are represented in Fig 9. The results of the walking tests due to two people for vertical vibrations and seven people for lateral vibrations are also shown in Fig 9, respectively. According to the results of the walking tests and the analysis, the accelerations of the walking tests are close to the results of analytical acceleration in pedestrian density in TC1.

<table>
<thead>
<tr>
<th>Comfort class</th>
<th>Degree of comfort</th>
<th>Acceleration for vertical</th>
<th>Acceleration for lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL 1</td>
<td>Maximum</td>
<td>&lt;0.50 m/sec$^2$</td>
<td>&lt;0.10 m/sec$^2$</td>
</tr>
<tr>
<td>CL 2</td>
<td>Medium</td>
<td>0.50-1.00 m/sec$^2$</td>
<td>0.10-0.30 m/sec$^2$</td>
</tr>
<tr>
<td>CL 3</td>
<td>Minimum</td>
<td>1.00-2.50 m/sec$^2$</td>
<td>0.30-0.80 m/sec$^2$</td>
</tr>
<tr>
<td>CL 4</td>
<td>Unacceptable discom fort</td>
<td>&gt;2.50 m/sec$^2$</td>
<td>&gt;0.80 m/sec$^2$</td>
</tr>
</tbody>
</table>

Table 3 : Defined comfort classes with common acceleration ranges.
According to Fig 9 (a), when pedestrian density is TC5 (1.5 \( P/m^2 \)), the maximum acceleration of the footbridge for vertical direction is not over CL4 (2.5 m/sec\(^2\)). According to Fig 9 (b), when pedestrian density is TC5 (1.5 \( P/m^2 \)), the maximum acceleration of the footbridge for lateral direction is in CL2 (0.1-0.3 m/sec\(^2\)). Therefore, the vibration serviceability for the object footbridge is satisfactory.

8 CONCLUSIONS

In order to grasp the natural frequencies and damping characteristics of the object footbridge, and in order to estimate the vibration serviceability performance due to pedestrians walking, this study conducted the examination and the analysis using the Hivoss guidelines for a prestressed concrete stress ribbon footbridge with external tendons and bearings. According to the results of the investigation, the vibration characteristics and serviceability of the bridge due to the pedestrians walking was confirmed.

The knowledge acquired by this study is as follows:

(1) Although a conventional type of stress ribbon footbridge has a bending asymmetry 1\(^{st}\) mode as the first vertical vibration mode, in this bridge, the first vertical vibration mode appeared as bending symmetry 1\(^{st}\).

(2) Two vertical vibration modes (Bending symmetry 2\(^{nd}\) and Bending asymmetry 2\(^{nd}\)) and a lateral vibration mode (lateral-torsion 1\(^{st}\)) existed in the critical ranges. The results of the analytical frequencies were close to the results of the experiment.

(3) Each member’s equivalent damping constants were evaluated by the GA (Genetic Algorithm) method using the experimental damping constants and the ratio of the strain energy. The results are as follows: stress ribbon deck 0.018; external tendon 0.007; struts and lateral member 0.04; and rubber bearing 0.093. Also, the results of the analytical strain energy proportional damping constants are close to the results of the experimental damping constants.

(4) According to Fig 9 (a), when pedestrian density is TC5 (1.5 \( P/m^2 \)), the maximum acceleration of the footbridge for vertical direction is not over CL4 (2.5 m/sec\(^2\)). According to Fig 9 (b), when pedestrian density is TC5 (1.5 \( P/m^2 \)), the maximum acceleration of the footbridge for lateral direction is in CL2 (0.1-0.3 m/sec\(^2\)). Therefore, the vibration serviceability for the object footbridge is satisfactory.

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