Time Domain Identification of the Tensile Force in Metallic Tie-Rods

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ABSTRACT: In this paper a method is presented in order to experimentally identify the tensile force in tie-rod of monumental masonry buildings. The method requires: (i) the modellation of the tie-rod masonry system as an Euler-Bernoulli beam with two internal spring-hinged devices; (ii) the identification of the tensile force by the knowledge of the dynamic response in time domain.

1 INTRODUCTION

The insertion of tie-rods into the vaults of brick or stone masonry structures is a technique that has been widely adopted since antiquity to eliminate the thrust exercised by the vaults on the elements sustaining them. It follows that the metallic tie-rod give a fundamental contribution to the structural equilibrium. Therefore, in order to guarantee the safety of the ancient masonry building the knowledge of the tensile forces becomes necessary.

The approaches currently adopted to determine the axial stress present in the existing tie-rods require two main steps: (i) the definition of an appropriate mechanical/numerical model to analytically treat the tie-rod masonry wall (TRMW) system; (ii) the definition of a procedure to experimentally identify the tensile force in the tie-rod.

About the former item, until few years ago the tie-rod was modeled as a vibrating wire, neglecting their bending stiffness. This model is acceptable only if the slenderness of the tension rod is high. However, in many actual cases both chains' section and devices, anchoring them in the surrounding masonry, are such that the bending stiffness cannot be neglected. In literature several more accurate models have been introduced to simulate the behaviour of the tie-rods. In particular, the TRMW system has been often modeled as an Euler-Bernoulli beam of uniform cross-section which has been assumed to be simply supported at the ends with additional rotational springs (Sorace (1996), Briccoli Bati and Tonietti (2001), Lagomarsino and Calderini (2005)). More recently, Amabili et al. (2010) modeled the TRMW system as a simply supported Timoshenko beam with the supports, inside the masonry wall, subjected to an elastic Winkler foundation simulating the interaction between the beam and the wall.

Until a few years ago, the tensile force in actual tie-rod was evaluated considering the tie-rods as a vibrating wire and measuring its first modal frequency. Although this methodology is still broadly diffused in the common practice, recently new identification methods have been developed. These methods are based on non-destructive static or dynamic tests or on a combination of both. In particular, Sorace (1996) proposed combined static-dynamic tests. Briccoli Bati and Tonietti (2001) introduced a single static test to identify the tensile force. The test requires the measurement of three vertical displacements under a concentrated static load, and the strains variations at three sections of the rod. The algorithm developed by Lagomarsino and Calderini (2005) identifies the axial tensile force by using the first three natural frequencies of the tie rod. Amabili et al. (2010) proposed a frequency based identification method that allows to minimize the measurement error and that is of simple execution.

In the framework of dynamic approaches, in the present paper a bounded variables time domain least square identification procedure is proposed. The basic idea of the least square method consists in the minimization of the so called penalty function, representing the squared difference between selected measured response parameters and the correspondent quantities determined through the study of a pertinent model.

The proposed procedure requires the following steps: (i) the adoption, for the TRMW system, of an uniform Euler-Bernoulli beam model with two internal spring-hinged devices, in order to write the differential equations governing the problem; (ii) the application of the weighted residuals method, to discretize the problem; (iii) the determination of the differential equations governing the response sensitivity evolution; (iv) the use of a numerical method, to solve both the generalized differential equations of motion and the sensitivity differential equations; (v) the minimization of the penalty function.

For validating the proposed technique laboratory tests have been conducted on metallic tie-rods.

2 THEORETICAL MODEL

The structural model employed is illustrated in Fig. 1. The tie-rod masonry wall (TRMW) system is assumed to be an undamped Euler-Bernoulli beam, with uniform section and with two internal hinges of rotational spring stiffness k_1 and k_2 at the positions z_1 and z_2, respectively, subjected to a constant axial tensile force T. Its transversal vibration $u_y(z,t)$ is governed by the partial differential equation:

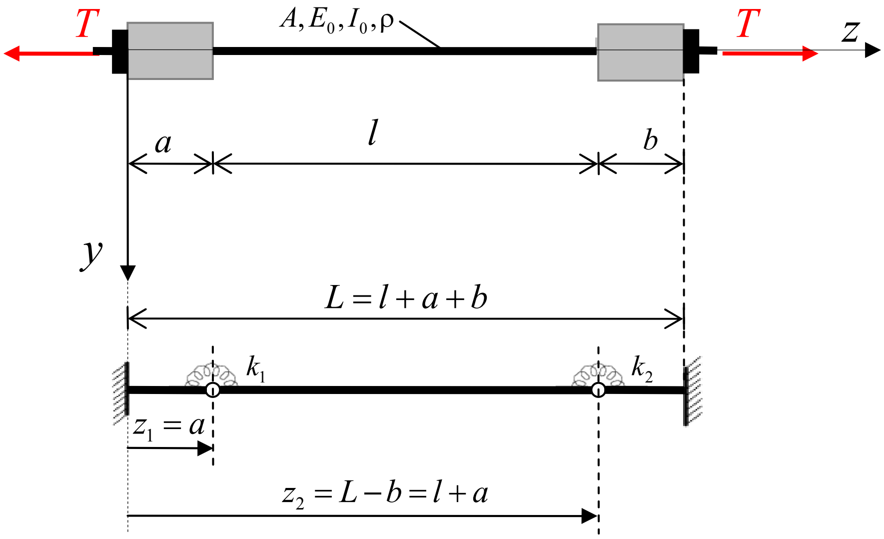

Figure 1. Schematic view of the adopted model

$$\frac{\partial^2}{\partial z^2}\left\{E_0\, I_x(z)\left[\frac{\partial^2 u_y(z,t)}{\partial z^2}\right]\right\} - T\frac{\partial^2 u_y(z,t)}{\partial z^2} + \rho\, A\frac{\partial^2 u_y(z,t)}{\partial t^2} = f_y(z,t); \quad 0<z<L \tag{1}$$

where $E_0\, I_x(z)$ is the bending stiffness, given by the product of the Young's modulus E_0 and the inertia moment of the cross-section $I_x(z)$; $\rho\, A$ is the mass per unit length, given by the product of mass density ρ and the cross-section area A; $f_y(z,t)$ is the external exciting force function.

According to the model proposed by Biondi and Caddemi (2005), two Dirac's delta singularities, $\delta(z)$, are introduced to model the flexural stiffness of the beam of Fig.1, with two internal hinges of rotational spring stiffness k_1 and k_2:

$$E_0\, I_x(z) = E_0\, I_0\left[1-\sum_{i=1}^{2}\beta_i\delta(z-z_i)\right]; \quad \beta_i=\frac{E_0\, I_0}{k_i + C\, E_0\, I_0} \qquad 0<z<L \tag{2}$$

where $C = 2.013$ is a dimensional parameter [length^{-1}].

Aim of this paper is to propose a procedure to experimental determine the unknown parameters: the tensile force T and the stiffness of the rotational springs k_1 and k_2. Notice that, even though $E_0 I_0$ might appear as a known parameter, in consideration of the ease in measuring the section of the tie-rod and of the stability of the elastic modulus of the iron, actually the complex situations of historical buildings makes its evaluation particularly difficult.

3 IDENTIFICATION PROCEDURE

In this section the main steps of the procedure recently proposed by Cacciola et al. (2010) to identify the unknown structural parameters of a discretized structure are summarized. In order to do this let us collect in the vector $\boldsymbol{\alpha}$ the p unknown structural parameters. The procedure is based on the minimization of penalty functions in a least square sense. The penalty function $J(\boldsymbol{\alpha})$ is defined by the following equation

$$J(\boldsymbol{\alpha}) = \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\varepsilon} \tag{3}$$

where the superscript T represents the transpose operation and

$$\boldsymbol{\varepsilon} = \delta\mathbf{y} - \mathbf{S}_{\boldsymbol{\alpha}_0}\,\delta\boldsymbol{\alpha} \tag{4}$$

In this Equation $\delta\mathbf{y} = \mathbf{y}^{(m)} - \mathbf{y}(\boldsymbol{\alpha}_0)$ is a vector which represents the difference between the measured response $\mathbf{y}^{(m)}$, evaluated in selected points of the beam, and the computed response quantities $\mathbf{y}(\boldsymbol{\alpha}_0)$, numerically evaluated in the same points in which the response is measured; furthermore $\mathbf{S}_{\boldsymbol{\alpha}_0}$ is the sensitivity matrix listing the partial derivative of the response $\mathbf{y}(\boldsymbol{\alpha})$ with respect the elements of the vector of the unknown structural parameters $\boldsymbol{\alpha}$ determined for an assigned initial set of parameters $\boldsymbol{\alpha}_0$ and $\delta\boldsymbol{\alpha} = \boldsymbol{\alpha} - \boldsymbol{\alpha}_0$. This matrix of order $r \times p$, with r the order of the vector of the measured response and p the number of unknown parameters, can be defined as

$$\mathbf{S}_{\boldsymbol{\alpha}_0} = \left.\left[\frac{\partial}{\partial \alpha_1}\mathbf{y}(\boldsymbol{\alpha}) \quad \cdots \quad \frac{\partial}{\partial \alpha_p}\mathbf{y}(\boldsymbol{\alpha})\right]\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} \tag{5}$$

The vector $\boldsymbol{\alpha}$ can be evaluated by the first order Taylor expansion of the structural response as a function of the unknown parameters $\boldsymbol{\alpha}$; namely by solving the following algebraic equation

$$\delta\mathbf{y} = \mathbf{S}_{\boldsymbol{\alpha}_0}\,\delta\boldsymbol{\alpha} \tag{6}$$

This equation can be straightforwardly solved only if the sensitivity matrix is square and not singular. Otherwise, the system will be over-determined (or under-determined) if the number of equations is greater: i.e. $p>r$ (or lesser: i.e. $p<r$) than the number of unknowns, respectively. In the case in which the sensitivity matrix is rectangular the solution of Eq.(6) can be achieved through a least square estimation by the means of the Moore-Penrose pseudoinverse (Friswell and Mottershead (1995)):

$$\boldsymbol{\alpha}_{j+1} = \boldsymbol{\alpha}_j + \left[\mathbf{S}^{\mathrm{T}}_{\boldsymbol{\alpha}_j}\mathbf{S}_{\boldsymbol{\alpha}_j}\right]^{-1}\mathbf{S}^{\mathrm{T}}_{\boldsymbol{\alpha}_j}(\mathbf{y}^{(m)} - \mathbf{y}(\boldsymbol{\alpha}_j)) \qquad j = 0,1,... \tag{7}$$

where $\boldsymbol{\alpha}_j$ is the vector of the estimated structural parameters at the j-th iteration, while $\mathbf{y}^{(m)}$ represents the vector of the measured response quantities. It is noted that in the case in which the number of unknowns exceeds the number of measured data the matrix $\mathbf{S}^{\mathrm{T}}_{\boldsymbol{\alpha}_j}\mathbf{S}_{\boldsymbol{\alpha}_j}$ is rank deficient, therefore alternative solutions have to be introduced (Friswell and Mottershead (1995)).

In this paper, according to the approach proposed by Cacciola et al (2010), the measured displacement time histories are used in order to identify the unknown structural parameters. The

advantage of this choice is due to the fact that the number of equations will certainly be greater than the number of the unknown parameters, namely Eq.(7) can be applied directly. Therefore, by means of Eq.(7) the identification procedure requires the following steps:
1) selection of the measuring point to be included in the vector $\mathbf{y}^{(m)}$;
2) experimental evaluation of the pertinent vector $\mathbf{y}^{(m)}$ collecting the measured displacement time histories;
3) definition of the vector $\boldsymbol{\alpha}$, listing the structural parameters to be identified and set of the initial value $\boldsymbol{\alpha}_0$;
4) numerical evaluation of the vector $\mathbf{y}(\boldsymbol{\alpha}_0)$ along with pertinent sensitivity matrix;
5) model updating, i.e. evaluation of vectors $\boldsymbol{\alpha}_{j+1}$ by means of Eq.(7);
6) numerical evaluation of the vector $\mathbf{y}(\boldsymbol{\alpha}_{j+1})$ along with pertinent sensitivity matrix;
7) iteration step 5 until the convergence.

It should be noted that the evaluation of the sensitivity matrix along with its inverse is certainly the most challenging issue from a computational point of view in the described identification scheme. The next section is devoted to the evaluation of the response and the sensitivity.

4 STRUCTURAL RESPONSE AND SENSITIVITY

4.1 Structural response

According to the Galerkin method the lateral deflection $u_y(z,t)$ can be approximated by the following series:

$$u_y(z,t) = \sum_{k=1}^{m} \psi_k(z)\, q_k(t) = \boldsymbol{\psi}^{\mathrm{T}}(z)\,\mathbf{u}(t) \tag{8}$$

where $\psi_k(z)$ is a sequence of linearly independent functions satisfying the boundary conditions, $\boldsymbol{\psi}(x)$ is the vector, of order $m \times 1$, collecting the functions $\psi_k(z)$ chosen for the beam, while $\mathbf{u}(t)$ is the vector, of order $m \times 1$, listing the associated generalised coordinates

$$\boldsymbol{\psi}(z) = \begin{Bmatrix} \psi_1(z) \\ \psi_2(z) \\ \cdots \\ \psi_m(z) \end{Bmatrix}; \quad \mathbf{u}(t) = \begin{Bmatrix} q_1(t) \\ q_2(t) \\ \cdots \\ q_m(t) \end{Bmatrix} \tag{9}$$

By applying the coordinate transformation (8) to Equation of motion (1), pre-multiplying the result by $\boldsymbol{\psi}(z)$ and integrate over the beam length (0, L) the following equation is obtained:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{K}\,\mathbf{u}(t) = \mathbf{f}(t) \tag{10}$$

where the dot over a variable denotes total derivative with respect to time t. The symmetric matrices $\mathbf{M}$ and $\mathbf{K}$, and the forcing vector $\mathbf{f}(t)$, are defined as

$$\begin{aligned} \mathbf{M} &= \rho A \int_0^L \boldsymbol{\psi}(z)\boldsymbol{\psi}^{\mathrm{T}}(z)\mathrm{d}z; \\ \mathbf{K} &= E_0 \int_0^L I_x(z)\boldsymbol{\psi}''(z)\boldsymbol{\psi}''^{\mathrm{T}}(z)\mathrm{d}z + T\int_0^L \boldsymbol{\psi}'(z)\boldsymbol{\psi}'^{\mathrm{T}}(z)\mathrm{d}z; \\ \mathbf{f}(t) &= \int_0^L \boldsymbol{\psi}(z) f_y(z,t)\,\mathrm{d}z. \end{aligned} \tag{11}$$

where the apex denotes total derivative with respect to spatial variable z. By assuming as trial functions the admissible functions of a clamped-clamped beam:

$$\psi_g(z) = 1 - \cos\left(2\frac{g\pi}{L}z\right); \tag{12}$$

the elements K_{gh} and M_{gh} of the symmetric matrices $\mathbf{M}$ and $\mathbf{K}$ can be evaluated in closed form:

$$\begin{aligned}
K_{gg} &= \left(\frac{2g\pi}{L}\right)^2 \left[\frac{TL}{2} + E_0 I_0 \left(\frac{2g\pi}{L}\right)^2 \left[\frac{L}{2} - \sum_{i=1}^{2} \beta_i \cos^2\left(\frac{2g\pi}{L} z_i\right)\right]\right];\\
K_{gh} &= K_{hg} = -E_0 I_0 \left(\frac{2g\pi}{L}\right)^2 \left(\frac{2h\pi}{L}\right)^2 \sum_{i=1}^{2} \beta_i \cos\left(\frac{2g\pi}{L} z_i\right)\cos\left(\frac{2h\pi}{L} z_i\right), \quad \forall\, g \neq h;\\
M_{gg} &= \frac{3\rho AL}{2};\\
M_{gh} &= M_{hg} = \rho AL, \quad \forall\, g \neq h;
\end{aligned} \tag{13}$$

4.2 Structural sensitivity

It has been assumed that the parameters to be identified are collected in the *p*-order vector α, p being the number of the significant parameters taken into account. Accordingly, Eq.(10) is re-written as

$$\mathbf{M}\,\ddot{\mathbf{u}}(\alpha,t) + \mathbf{K}(\alpha)\,\mathbf{u}(\alpha,t) = \mathbf{f}(t) \tag{14}$$

where the dependence of the mass matrix, loading vector and response vector on α has been introduced.
In order to evaluate the *k*-th sensitivity vector, the equation of motion (19) is cast in the form of a dynamically modified linear system (Muscolino (1996)) as follows

$$\mathbf{M}\,\ddot{\mathbf{u}}(\alpha,t) + \left[\mathbf{K}_{\alpha_j} + \Delta\mathbf{K}(\alpha)\right]\mathbf{u}(\alpha,t) = \mathbf{f}(t) \tag{15}$$

in which $\mathbf{K}_{\alpha_j}$ is the mass matrix of the structure evaluated in correspondence of the parameter vector α_j, while $\Delta\mathbf{K}(\alpha) = \mathbf{K}(\alpha) - \mathbf{K}_{\alpha_j}$. The time-variability of the sensitivity vector is governed by a set of second-order differential equations obtained by differentiating Eq.(14) with respect to α_k, obtaining

$$\mathbf{M}\,\ddot{\mathbf{s}}_{\mathbf{u}_i,k}(\alpha_j,t) + \left[\mathbf{K}_{\alpha_j} + \Delta\mathbf{K}(\alpha_j)\right]\mathbf{s}_{\mathbf{u}_i,k}(\alpha_j,t) = \mathbf{K}_k^I(\alpha_j)\,\mathbf{u}(\alpha_j,t) \tag{16}$$

where

$$\mathbf{s}_{\mathbf{u},k}(\alpha_j,t) = \left[\frac{\partial}{\partial\alpha_k}\mathbf{u}(\alpha,t)\right]\Bigg|_{\alpha=\alpha_j}; \quad \mathbf{K}_k^I(\alpha_j) = \frac{\partial}{\partial\alpha_k}\mathbf{K}(\alpha)\Bigg|_{\alpha=\alpha_j} \qquad k = 1,\ldots,p \tag{17}$$

Since the last term in Eq.(17) is a known quantity, it can be considered as forcing terms in the solution of this equation. Thus, the mathematical structure of Eq.(17) is formally similar to the one of Eq.(14), which represents the equation of motion of the structural system. Eq. (14). It follows that both differential equations can be integrated by standard step-by-step algorithms. In this paper, Newmark-β method (β =1/6, γ=1/2) is adopted.

4.3 Sensitivity matrix

In order to apply the iterative procedure (7) the vectors $\mathbf{y}^{(m)}$ and $\mathbf{y}(\boldsymbol{\alpha}_j)$, listing respectively the measured and numerically evaluated response for the estimated structural parameters at the *j*-th iteration $\boldsymbol{\alpha}_j$, have to be known. These vectors are defined as follows

$$\mathbf{y}^{(m)} = \begin{Bmatrix} w(z^{(k)}, \Delta t) \\ \vdots \\ w(z^{(k)}, n_t \Delta t) \end{Bmatrix} \qquad \mathbf{y}(\boldsymbol{\alpha}_j) = \begin{Bmatrix} w(z^{(k)}, \boldsymbol{\alpha}_j, \Delta t) \\ \vdots \\ w(z^{(k)}, \boldsymbol{\alpha}_j, n_t \Delta t) \end{Bmatrix} \tag{18}$$

where $w(z^{(k)}, \ell\Delta t)$ is the measured response (which could be evaluated in terms of displacement or acceleration) determined at the position $z^{(k)}$ at the ℓ-th time instant; $w(z^{(k)}, \boldsymbol{\alpha}_j, \ell\Delta t)$ is the numerically evaluated displacement response evaluated at the *j*-th iteration in the position $z^{(k)}$ at the ℓ-th time instant; Δt is the sampling time; $n_t \Delta t = t_f$ is the duration of the recording and n_t is the number of time steps used. It follows that the vectors $\mathbf{y}^{(m)}$ and $\mathbf{y}(\boldsymbol{\alpha}_j)$ are of order $r \times 1$, with $r \equiv n_t$. From a computational point of view the vector $\mathbf{y}(\boldsymbol{\alpha}_j)$, by applying the Galerkin method, can be suitably determined as follows:

$$\mathbf{y}(\boldsymbol{\alpha}_j) = \begin{Bmatrix} \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{u}(\boldsymbol{\alpha}_j, \Delta t) \\ \vdots \\ \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{u}(\boldsymbol{\alpha}_j, n_t \Delta t) \end{Bmatrix} \text{ or } \mathbf{y}(\boldsymbol{\alpha}_j) = \begin{Bmatrix} \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\ddot{\mathbf{u}}(\boldsymbol{\alpha}_j, \Delta t) \\ \vdots \\ \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\ddot{\mathbf{u}}(\boldsymbol{\alpha}_j, n_t \Delta t) \end{Bmatrix} \tag{19}$$

where $\boldsymbol{\psi}(z^{(k)})$ is the vector, of order $m \times 1$, defined in Eq.(9) evaluated at the position $z = z^{(k)}$.

In order to apply the iterative identification procedure described in section 3, the sensitivity matrix $\mathbf{S}_{\boldsymbol{\alpha}_j}$ has to be evaluated. The *p* columns of this, taking into account Eqs. (5) and (19), can be written in the following form, respectively:

$$\mathbf{S}_{\boldsymbol{\alpha}_j} = \begin{bmatrix} \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\mathbf{u},1}(\boldsymbol{\alpha}, \Delta t) & \cdots & \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\mathbf{u},p}(\boldsymbol{\alpha}, \Delta t) \\ \vdots & & \vdots \\ \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\mathbf{u},1}(\boldsymbol{\alpha}, n_t \Delta t) & \cdots & \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\mathbf{u},p}(\boldsymbol{\alpha}, n_t \Delta t) \end{bmatrix}_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_j} \tag{20}$$

or

$$\mathbf{S}_{\boldsymbol{\alpha}_j} = \begin{bmatrix} \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\ddot{\mathbf{u}},1}(\boldsymbol{\alpha}, \Delta t) & \cdots & \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\ddot{\mathbf{u}},p}(\boldsymbol{\alpha}, \Delta t) \\ \vdots & & \vdots \\ \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\ddot{\mathbf{u}},1}(\boldsymbol{\alpha}, n_t \Delta t) & \cdots & \boldsymbol{\psi}^{\mathrm{T}}(z^{(k)})\mathbf{s}_{\ddot{\mathbf{u}},p}(\boldsymbol{\alpha}, n_t \Delta t) \end{bmatrix}_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_j} \tag{21}$$

where the sensitivity vector $\mathbf{s}_{\mathbf{u},k}\left(\boldsymbol{\alpha}_j, \ell\Delta t\right)$, $\ell = 1,\ldots,n_t$; $k = 1,\ldots,p$, has been defined in Eq.(17) and $\mathbf{s}_{\ddot{\mathbf{u}},k}\left(\boldsymbol{\alpha}_j, \ell\Delta t\right)$ is the sensitivity vector in terms of acceleration.

5 NUMERICAL APPLICATION

The feasibility of the proposed least square identification procedure is demonstrated using the laboratory testing data from a metallic bar subjected to the tensile force of 20.1 kN. Fig. 2 shows the bar utilized in the testing along with the supporting structure.

Figure 2. A metallic-rod used in the laboratory experimental

The 2.5 cm diameter metallic-rod with the length of 3.07 m, threaded at its ends, is connected at one end to a load cell and at the other to a doubly bolted support anchored to a highly rigid steel frame. This special experimental arrangement permittes varying the tie-rod tensioning measured by the load cell.

Fixed response measurement testing with roving hammer impacts is performed to collect dynamic response data. Two accelerometers are located at a distance equal to 1 m from either end point of the rod. Instrumentation used to conduct the test consists of an impulse hammer Dytran 9.9 mV/g, two accelerometers Dytran 9.9 mV/g, a data acquisition system HBM MCG Plus and a portable computer.

Aim of this section is the identification of tensile force T assuming two identical rotational springs at both ends of the model. In this case we have only one unknown parameter ($\alpha \equiv T$) and the matrix at j-th iteration, $\mathbf{K}_k^I\left(T_j\right)$, becomes a diagonal one with elements:

$$K_{k,gg}^I\left(T_j\right) \equiv K_{k,gg}^I = \left(\frac{2g\pi}{L}\right)^2 \frac{L}{2} \tag{22}$$

The material properties assigned to the structure are the mass density $\rho = 7800 \text{ kg/m}^3$ and the measured elastic modulus $E_0 = 205 \times 10^9 \text{ N/m}^2$.

The measured response quantity $\mathbf{y}^{(m)}$ here represents the historical recorded acceleration generated by the hammer impact shown in Fig. 3.

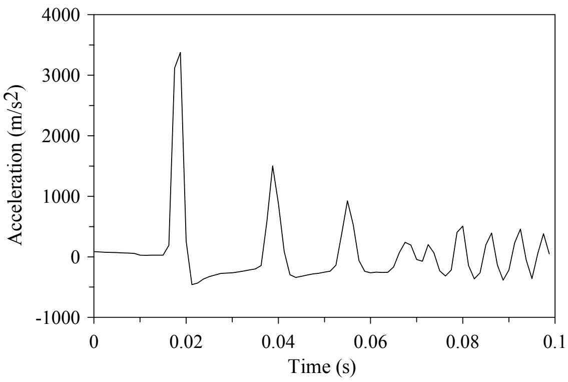

Figure 3. Time-domain impact hammer

The computed response quantity $\mathbf{y}(\alpha_0)$ is evaluated trough numerical simulation on the discretized model starting by an hypothetical initial value $\alpha_0 = T$ of the tensile force. Table 1 reports the convergency of the proposed method for several initial values of the tensile force.

Case	Number of iterations	Tensile Force (kN) Initial value	Target	Identified	Error (%)
1	9	10.00	20.10	20.04	0.28
2	6	15.00	20.10	20.04	0.28
3	7	25.00	20.10	20.04	0.28
4	9	30.00	20.10	20.04	0.28
5	16	35.00	20.10	20.04	0.28

For validating the adopted structural model, the frequencies of the metallic rod have been calculated ($\omega_{i,c}$) through an eigenproblem and compared to the measured ones. Fig. 4 shows the comparison between the first three frequencies since the highest measured frequencies can be affected by possible errors.

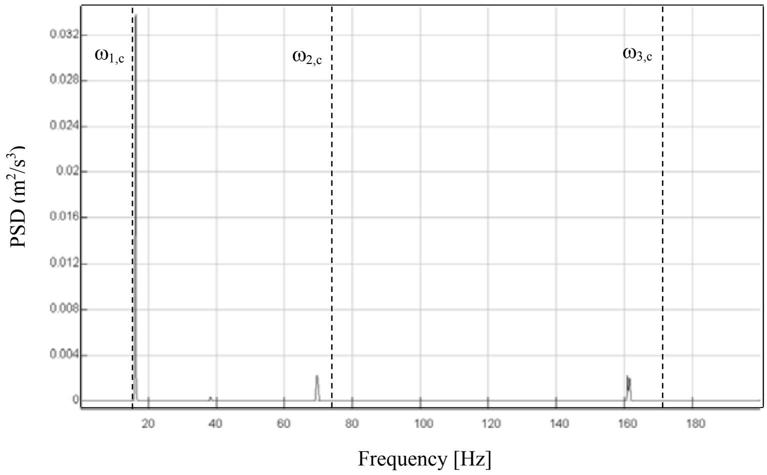

Figure 4. Comparison between measured and calculated (dashed-line) frequencies

6 CONCLUSIONS

In the framework of the time domain parametric identification techniques a bounded least square procedure, based on the minimization of a postulated penalty function, is proposed in this paper. The procedure minimizes the difference between measured and calculated acceleration time-histories of a tie-rods modelled as an Euler-Bernoulli beam with two internal spring-hinged devices. In this paper a method is presented in order to experimentally identify the tensile force in tie-rod of monumental masonry buildings. Once the material properties are noted, the proposed procedure allows to identify the unknown tensile force by starting with a tentative value. For validating the technique laboratory tests have been conducted on a metallic tie-rod.

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