NONLINEAR MODAL IDENTIFICATION BY USING TIME AND FREQUENCY FORCED RESPONSES

RAYNAUD J.L., Femto-st -Besançon France

FOLTETE E, Femto-st -Besançon France

PERRIOT A., Femto-st -Besançon France

PIRANDA J., Femto-st -Besançon France

jean-louis.raynaud@univ-fcomte.fr

Abstract

In this article, a method of modal identification for weakly non-linear structures is presented based on the time and frequency responses resulting from a random or sine sweep excitation. After extracting the contributions of distant modes, the formulation is based on the hypothesis of the first harmonic by eliminating the perturbations due to higher harmonics in the responses. We then condense the information derived from the set of "virtual" pickups corresponding to the number of simultaneously identified modes, coupled or not. A technique of equivalent linearization allows the linear and non-linear terms, which are taken into account by the identification procedure, to be determined.

1 Introduction

During the last few years, many studies have tried to take into account nonlinearities contributions in the dynamical behaviour of mechanical structures. Two different approaches can be distinguished either based on the time responses or the frequency responses. The industrial structures often present a nonlinear behaviour and the distortions caused by the nonlinearities have to be taken into account in order to identify the real dynamic of the structures.

In the case of isolated modes in the frequency spectrum, simple methods are extended from linear modal analysis techniques. The problem is more complex when several heavily coupled modes exist.

In the time domain, Feldman and Braun (1997), Huang and Iwan (1997) propose methods based on the Hilbert transform and well-adapted for structures having a strong nonlinear behaviour. Concerning the cas of weak nonlinearities, several authors can be quoted. Dippery and Weaver Smith (1998) propose a multi degrees of freedom method based on the "minimal model error". Staszewski and Chance (1997) apply a wavelet theory for a single mode structure. Atkins and Worden (1996) use a "Direct Parameter Estimation" and a detection criterion with a two degrees of freedom example.

All those different methods show that studying nonlinear structures is complex. Few of them deal with coupled modes because the number of parameters to be identified increase dramatically.

The great interest of the new method proposed here is to decrease the number of degrees of freedom to the number of modes existing in the analysed frequency band. It is based on a condensation of the measured forced responses. The position and type of the nonlinearities can be unknown. Their effects on the structure responses are taken into account by cubic damping and stiffness terms. This method can be used either in the time domain or in the frequency domain.

2 The proposed method

Time or frequency responses recorded on the structure are used. They are due to a chosen excitation (random, swept sine, stepped sine, …). Two modes are considered as coupled modes if the difference between their respective eigenfrequency is less than two time the -3dB bandwidth of the most damped mode.

The excitation force f is controlled such as its spectrum only contains frequencies in the vicinity of the eigenfrequencies which must be identified. The first harmonic hypothesis is used, and only the Fourier decomposition terms of the time responses which are inside the excitation frequency band are kept.

The structure is discretized by N degrees of freedom (dof).

In the time domain, the responses y(t) verify:

$$M\ddot{y}(t)+(B+B_n)\dot{y}(t)+(K+K_n)y(t)=f(t) \tag{1}$$

In the frequency domain, the responses y(ω) verify:

$$M\ddot{y}(\omega)+(B+B_n)\dot{y}(\omega)+(K+K_n)y(\omega)=f(\omega) \tag{2}$$

where:

1. The N order matrices M, B, K are respectively the mass, damping and stiffness matrices, which are real, symmetric, positive definite and constant ($\in R^{N,N}$).
2. The N order matrices B_n and K_n are the nonlinear damping and stiffness matrices. They are symmetric, time dependant (time domain) or frequency dependant (frequency domain) and vary with the relative displacement amplitude between the degrees of freedom.

c sensors and e exciters are placed on the structure. The response Y_k of the sensors due to a force configuration F_k is recorded. The size of Y_k is (c,p) where p is the number of time or frequency records. If e different force configurations are applied, the corresponding responses and forces can be assembled into the matrices Y and F respectively:

$$Y=[Y_1 \quad Y_2 \quad \dots \quad Y_e], \qquad F=[F_1 \quad F_2 \quad \dots \quad F_e] \tag{3}$$

2.1 Contribution of the out-of-band modes (frequency domain)

If the responses are measured in the frequency domain, the contribution of the out-of-band modes can be attenuated by using a difference technique. For a given frequency increment δp, the point-to-point difference ΔY_k is defined by:

$$\Delta Y_{k\,ij} = Y_{k\,i}(\omega_j+\delta p)-Y_{k\,i}(\omega_j) \tag{4}$$

As the out-of-band modes contribution varies slowly, it is strongly reduced in the difference matrix ΔY defined by:

$$\Delta Y = [\Delta Y_1 \quad \Delta Y_2 \quad \ldots \quad \Delta Y_e] \tag{5}$$

2.2 Determination of the condensation matrix

The records are arranged in the matrix Y_0 such that:

1. for time responses:

$$Y_0 = Y \tag{6}$$

2. for frequency responses:

$$Y_0 = [\text{réel}(\Delta Y), \text{imag}(\Delta Y)] \tag{7}$$

In order to identify the m coupled modes existing in the analysed frequency band, a real basis U_1 representing the whole vectors of the matrix Y_0 with a minimal rank is searched. That basis is obtained by using a Singular Value Decomposition:

$$Y_0 = US^T V \tag{8}$$

The rank of the matrix Y_0 allows identifying the number of modes m. The matrix U_1 is constituted by the m first columns of the matrix U and describes the subspace defined by the m concerned modes.

By projecting Y_0 over U_1, the condensed responses are obtained:

$$C = {}^T U_1 Y_0 \tag{9}$$

C can be considered as the structure response over m virtual sensors. Due to its reduced size, the determination of the linear and nonlinear parameters will be easier.

2.3 Principle of the condensation

Let T be the condensation matrix such that for each instant t (time domain) or for each circular frequency ω (frequency domain):

$$y = Tc \tag{10}$$

The matrix T can be written as:

$$T = \begin{bmatrix} U_1 \\ U_i \end{bmatrix} \tag{11}$$

where:

1. U_1 is a known matrix of size (m,m).

2. U_i is an unknown matrix of size (N-m,m)

In the considered frequency band, the responses verify the relations (1) or (2). By using the previous expression (10) and after premultiplying by ${}^tT$, the relations (1) or (2) become:

$$m_0\ddot{c} + \left(b_0 + \tilde{b}\right)\dot{c} + \left(k_0 + \tilde{k}\right)c = f_0 \qquad (12)$$

with:

$$m_0 = {}^tTMT \qquad b_0 = {}^tTBT \qquad k_0 = {}^tTKT$$

$$\tilde{b} = {}^tTB_nT \qquad \tilde{k} = {}^tTK_nT \qquad f_0 = {}^tTf$$

A condensed model with m degrees of freedom is obtained, where the matrices m_0, b_0 et k_0 are symmetric, constant, positive definite, $\tilde{b}$ and $\tilde{k}$ are real matrices which represent the nonlinear behaviour of the structure.

3 Modelling the nonlinearities

In practice, the structural nonlinearities can have various and unknown origins. The experience has shown that the approximation of the nonlinear restoring forces by cubic functions is usually satisfying for a limited amplitude domain. For a stiffness nonlinearity between two degrees of freedom i and j, this approximated restoring force can be written:

$$f_{nl}^{i}(y) = -f_{nl}^{j}(y) = \xi_{ij}\left(y_i - y_j\right)^3 \qquad (13)$$

Where y_i and y_j are the unknown displacements at dof i and j respectively and ξ_{ij} is a real constant.

If T_i is the row number i of the matrix T, then:

$$y_i - y_j = T_{ij}c \qquad (14)$$

with: $T_{ij} =: T_i - T_j$

The relation (12) becomes:

$$m_0\ddot{c} + b_0\dot{c} + k_0 c + f_{nl}^{ij}(c) = f_0 \qquad (15)$$

The nonlinear force can be written in the condensation basis:

$$f_{nl}^{ij}(c) = \xi_{ij}(c)^3\, {}^tT_{ij} \qquad (16)$$

If nl nonlinearities exist between the dof couple (i,j), the total nonlinear force is expressed in the condensed problem by:

$$f_{nl}(c) = \sum_{(i,j)=1}^{nl} f_{nl}^{ij}(c) \qquad (17)$$

4 Identifying the model

The condensed nonlinear behaviour model is described by:

$$m_0\ddot{c} + b_0\dot{c} + k_0 c + f_{nl} = f_0 \tag{18}$$

The identification of the matrices m_0, b_0, k_0 and of the nonlinear terms is performed by solving a linear equivalent system.

The total number of unknown parameters is: $3C^2_{m+1} + 2C^4_{4+m+1}$.

For example, if the number of considered modes is between one and for:

- 1 mode: m=1 gives 5 parameters.
- 2 modes: m=2 gives 19 parameters.
- 3 modes: m=3 gives 48 parameters.
- 4 modes: m=4 gives 100 parameters.

The condensed nonlinear problem (12) can be transformed into an equivalent linear system (19) where the unknown vector x contains the condensed mass, damping, stiffness and the nonlinear parameters. The matrix A is built from the condensed responses and the right-hand vector b contains the condensed forces.

$$A\ x = b \tag{19}$$

The real eigenvalues and eigenvectors (ω_r, Q_r) of the conservative condensed problem are estimated by solving the homogeneous problem (20):

$$\left(k_0 - \omega_r^2 m_0\right) q_r = 0 \tag{20}$$

The real eigenvectors associated to the structure can then be obtained by:

$$Y_r = U_1 Q_r \tag{21}$$

The complex eigenvalues and eigenvectors (s_c, Q_c) of the dissipative condensed problem are given by solving the homogeneous problem (22):

$$\left(s_c^2 m_0 + s_c b_0 + k_0\right) q_c = 0 \tag{22}$$

Finally the complex eigenvectors associated to the structure are obtained by:

$$Y_c = U_1 Q_c \tag{23}$$

The quality of the identified nonlinear system can be estimated by comparing the measured forced responses to the synthesised ones in the frequency domain. A condensed synthesised response c_s is first computed by an iterative procedure based on relation (24). The iterations are stooped as soon as a convergence criterion is satisfied.

$$c_s(\omega) = \left(-\omega^2 m_0 + j\omega\omega_0 + k_0\right)^{-1}\left(f_0 - f_{nl}\left(c_s(\omega)\right)\right) \tag{24}$$

The structural forced response is then obtained by:

$$Y_s = U_1 Q_s \tag{25}$$

5 Numerical simulations

5.1 Frequency domain

In the frequency domain, the proposed method is evaluated on a simple 8 degrees of freedom system having 3 coupled modes, and with various number and type of nonlinearities. The modal parameters of the dissipative and conservative systems are very well estimated. Table n°1 shows the numerical results obtained for simultaneous cubic stiffness and damping nonlinearities at each dof.

Mode Number	Exact Frequency (Hz)	Estimated Frequency (Hz)	Exact damping coefficient (%)	Estimated damping coefficient (%)	MAC	MSF
1	10	9.9998	3.9789	3.9835	1.0000	1.0001
2	12	12.0001	3.3157	3.3177	1.0000	1.0002
3	14	14.0002	2.8421	2.8433	1.0000	1.0000

Table n°1: comparing results in the frequency domain

The estimated conservative system is compared to the exact one. The eigenfrequency errors are less than 2.10e-2 and the damping coefficient errors less than 10e-3. The real eigenvectors are compared using the Modal Assurance Criterion (MAC) and the Modal Scale Factor (MSF) defined by (26).

$$\mathrm{MAC}(x,y) = \frac{\left({}^{T}xy\right)^2}{\|x\|^2\|y\|^2} \quad \mathrm{MSF}(x,y) = \frac{{}^{T}xy}{\|y\|^2} \tag{26}$$

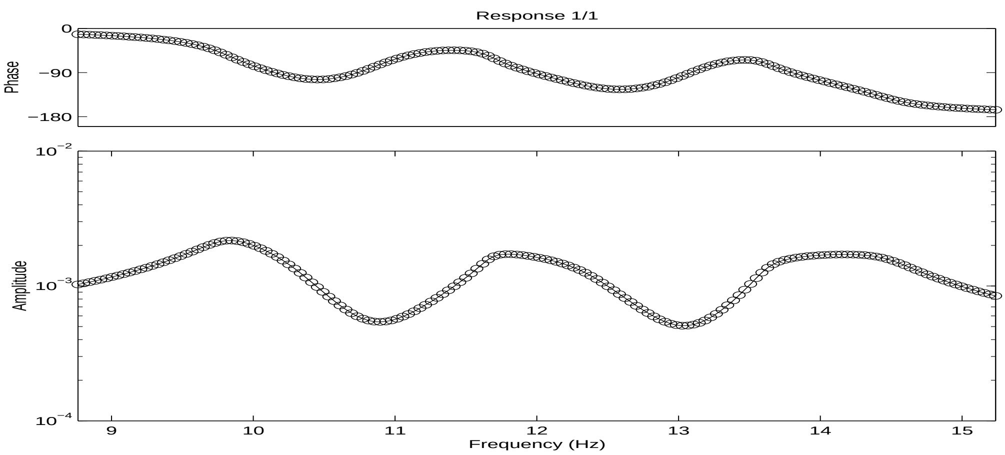

Figure 2 shows the exact (-) and synthesised (o) frequency response function on the first dof.

5.2 Time domain

The same 8 dof system has been used to simulate time responses to a swept sine excitation. The results are presented in table n°3 and figure n°4 shows an example of responses to this kind of excitation.

Mode Number	Exact Frequency (Hz)	Estimated Frequency (Hz)	Exact damping coefficient (%)	Estimated damping coefficient (%)	MAC	MSF
1	10	10.0006	3.9789	4.0939	1.0000	1.0003
2	12	11.9952	3.3157	3.3846	1.0000	0.9978
3	14	14.0213	2.8421	2.6151	1.0000	0.9953

Table n°3: comparing results in the time domain

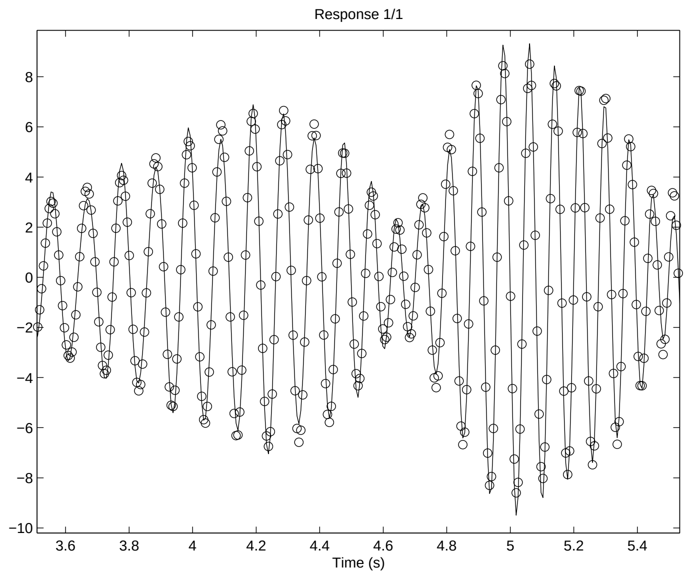

Figure 4 shows the exact (-) and synthesised (o) time response on the first dof

6 Conclusion

A new identification method for structures with a nonlinear behaviour has been presented. The forced responses are condensed over a number of virtual degrees of freedom corresponding to the number of modes effectively existing in the analysed frequency band. This condensation decreases significantly the number of parameters to be identified. This method has been validated on systems presenting simulated or experimental nonlinearities. Even if it is based on a cubic stiffness and damping modelling, it has been shown that it could take into account some other types of nonlinearities, such as square stiffness and damping or dry friction. In these last cases, the forced responses are not perfectly synthesised but the eigenmodes are much better identified than with a linear method.

7 References

[1] Atkins, P. and Worden, K. (1996) Identification of a Multi-Degree-Of-Freedom nonlinear system, *XIVth IMAC*, 1023-1028.

[2] Chong, Y.H. and Imregun, M. (1998) Modal Parameter Extraction methods for non-linear systems, *XVIth IMAC*, 728-736

[3] Dippery, K.D. and Weaver Smith, S. (1998) An optimal control approach to nonlinear system identification, *XVIth IMAC*, 637-643

[4] Feldman, M. and Braun, S. (1997) Description of free responses of SDOF systems via the phase plane and Hilbert transform : the concepts of envelope and instantaneous frequency, *XVth IMAC*, 973-979

[5] Ferreira, J.V. and Ewins, D.J. (1997) Algebraic nonlinear impedance equation using multi-harmonic describing function, *XVth IMAC*, 1595-1601

[6] Huang L. and Iwan, W.D. (1997) Approximate methods for calculation of nonlinear instantaneous mode shapes, *XVth IMAC*, 987-994

[7] Song, H.W. and Wang, W.L. (1998) Non-linear system identification using frequency domain measurement data, *XVIth IMAC*, 746-752

[8] Staszewski, W.J. and Chance, J.E. (1997) Identification of nonlinear systems using wavelets - Experimental study, *XVth IMAC*, 1012-1016

[9] Wardle, R., Worden, K., and King, N.E. (1997) Classification of nonlinearities using neural network, *XVth IMAC*, 980-986

[10] Weiland, M. and Link, M. (1996) A Direct Parameter Estimation method for weak nonlinear systems, *XIVth IMAC*, 525-531