

Load Spectrum Identification from Operative Responses

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Abstract

Operational forces have in general a stochastically nature and may depend on the structural behavior, as in aeroelastic problems. Generally speaking, such forces could not be directly measured by the common available techniques, due to the problems related to the measurement itself as accessibility, temperature, etc. In this paper, a methodology for an indirect identification of the load spectrum acting on structures, considered under the actual operative conditions, is presented. The developed approach, based on the method which allows one to estimate biased frequency response functions from the output only data, permits to solve the inverse problem in the frequency domain. Therefore, the load spectrum could be estimated, through the output responses, by identifying the modal parameters: natural frequencies, damping ratios, residues, eigenvectors and modal masses. The proposed methodology has been investigated on few simple experimental tests.

1 Introduction

The request of more and more efficient modal testing techniques, maintaining an accuracy adequate for a correlation with finite element analysis models - *i.e.*, model updating, load analysis, acoustic radiation, system synthesis, etc. - is one of the nowadays main target, in which product development and design refinement teams are interested. Those approaches, that only require the evaluation of the structure output responses, could be considered as a critical know-how development, which have to be considered by engineers. Among these techniques, which permit to get an accurate dynamic identification of the structure (with respect to the dynamic loads and boundary conditions), the approaches which use operational data are of great interest. In fact, it is difficult or even impossible to simulate the true working conditions in tests carried out on a structure in a laboratory. In addition, this kind of modal testing is achieved with a reduced time-to-market with consequent reduced costs.

Estimation methods are based on both time, Ref. [1], and frequency domains, Ref. [2]. In the frame of the frequency domain approaches, it has been shown, Refs. [3, 4] and Ref. [5], that a complete biased frequency response function matrix, *FRFs*, could be directly derived from the power spectral density matrix of the output responses taking advantages of the Hilbert transform properties. The main hypothesis of such approach is on the input loading that should be almost constant, at least in the band of interest.

The noise standard deviation represents the unknown biasing constant affecting the estimate of the *FRFs*. This approach requires first to estimate the phases of the diagonal terms from the auto-power spectral densities by applying the Hilbert transform, Ref. [6], so as to achieve the biased *FRFs*, and then to get the out-of-diagonal functions. Finally, the scaling factors could be estimated from two test configurations using the approach developed in [7, 8, 9], that - in turn - could be originated (for some items) from the pioneer work presented in Ref. [10]. In this approach, the generalized parameters could be derived from the evaluation of the eigenfrequency shift corresponding to known masses added at known structural locations. It is worth noting that such scaling factors are identified making use of operational data only and they do require neither the knowledge of a detailed mathematical model of the structure (*i.e.*, finite element model) nor restrictive assumptions regarding to the excitation signal.

In this paper, the developed procedure is pushed forward for estimating the load spectrum exciting the structure. This kind of inverse problem could be solved once the modal parameters and the relative modal masses are disposable. The assessment of the proposed approach is carried out considering experimental tests on simple structures.

2 Theoretical background

2.1 Biased FRF estimates by the *Hilbert* transform

The theory presented in Ref. [3] is, for the sake of simplicity of the reader, reported hereafter. When the output responses, at some points of the vibrating structure, are only available, their auto power spectral densities can be obtained. If we consider the output response, $x_i(t)$, at the i -th degree of freedom of the structure, the auto power spectral density $G_{x_i x_i}$ is given by the following relationship:

$$G_{x_i x_i}(\omega) = G_{f_i f_i}(\omega) |H_{ii}(\omega)|^2 \quad (1)$$

where $G_{f_i f_i}(\omega)$ is the auto power spectral density of the random input signal. If $G_{f_i f_i}(\omega)$ can be assumed constant in the frequency band of interest, taking the natural logarithm of Eq. 1, the Hilbert transform (\mathcal{H}) applied to $\ln |H_{ii}(\omega)|$ provides the phase, $\Phi_{ii}(\omega)$, between the output degrees of freedom :

$$\mathcal{H} [\ln G_{x_i x_i}(\omega)] = 2 \mathcal{H} [\ln |H_{ii}(\omega)|] = -2 \Phi_{ii}(\omega) \quad (2)$$

since the Hilbert transform of a constant is zero and the signals are causal, Refs. [6, 11]. Equations 1 and 2 can be used to estimate the frequency response function:

$$H_{ii}(\omega) = |H_{ii}(\omega)| e^{-j \frac{1}{2} \mathcal{H} [\ln G_{x_i x_i}(\omega)]} \quad (3)$$

The biased frequency response function is then achieved by introducing the relationship Eq. 1 into Eq. 3:

$$\tilde{H}_{ii}(\omega) = \sqrt{G_{f_i f_i}} H_{ii}(\omega) \quad (4)$$

So the diagonal terms of the *FRF* matrix are known except for a constant. Obviously, this does not affect the estimates of the modal parameters gained from the above mentioned frequency functions. In fact, the poles of the dynamic system are independent

from the (unknown) excitation level. If the first M modes of the structure have been excited, the driving point FRF s could be expressed as:

$$H_{ii}(\omega) = \sum_{k=1}^M \frac{R_{ii}^k \omega_{d_k}}{(\sigma_k + j\omega)^2 + (\omega_{d_k})^2} \quad (5)$$

where, for the k -th mode, σ_k is the decay rate, ω_{d_k} is the damped circular frequency, and R_{ii}^k is the residue. The modal parameters, *i.e.*, the natural frequencies, the damping ratios and the biased residues

$$\tilde{R}_{ii}^k = \frac{\sqrt{G_{f_i f_i}}}{m_k \omega_{d_k}} \quad (6)$$

can be identified.

Introducing the residue, defined in Eq. 6, into the relationship of Eq. 5, the biased frequency response function is gained:

$$\tilde{H}_{ii}(\omega) = \sum_{k=1}^M \frac{\tilde{R}_{ii}^k \omega_{d_k}}{(\sigma_k + j\omega)^2 + (\omega_{d_k})^2} \quad (7)$$

Once these frequency response functions have been estimated, the out of diagonal terms of the FRF matrix can be obtained as follows:

$$\tilde{H}_{ij}(\omega) = \frac{G_{x_i x_j}(\omega)}{\tilde{H}_{ii}^*(\omega)} \quad (8)$$

So the frequency response function matrix can be derived only through the knowledge of the auto and cross power spectral densities of the output signals.

2.2 Estimate of the input load spectrum

From the FRF 's mentioned above it is possible to estimate not only the natural frequencies and the damping factors, but also the biased residues, wherefrom the eigenvectors are derived. Let us consider the k -th mode, the relationship between the residues of the mode and the relative components of the eigenvector, for almost real modes, can be written as follows:

$$\begin{Bmatrix} \tilde{R}_{11}^{(k)} \\ \tilde{R}_{12}^{(k)} \\ \vdots \\ \tilde{R}_{1N}^{(k)} \end{Bmatrix} = \frac{1}{m_k \omega_{d_k}} \phi_1^{(m)} \begin{Bmatrix} \phi_1^{(k)} \\ \phi_2^{(k)} \\ \vdots \\ \phi_N^{(k)} \end{Bmatrix} \quad (9)$$

The first component of the residue vector is:

$$\tilde{R}_{11}^{(k)} = \frac{1}{m^{(k)} \omega_{d_k}} \left(\phi_1^{(k)} \right)^2 \quad (10)$$

normalizing the eigenvector components with respect to $\phi_1^{(k)}$, the modal mass is given by:

$$m_k = \frac{1}{\tilde{R}_{11}^{(k)} \omega_{d_k}} \quad (11)$$

it is worth noting that the modal mass results to be biased as derived from the biased residue. On the contrary, the components of the eigenvector are unbiased, in fact they can be obtained by the residue ratios:

$$\phi_i^{(k)} = \frac{\tilde{R}_{1i}^{(k)}}{\tilde{R}_{11}^{(k)}} \quad (12)$$

Now, the modal mass is experimentally identified with the approach introduced in Ref. [10] and also recently used with success in output-only techniques, Refs. [7, 8, 9], by the relationship:

$$m_k = -\frac{1}{2} \left(\frac{\omega_{n_k}}{\Delta\omega_k} \right) \sum_i^M \left(\phi_1^{(i)} \right)^2 \Delta m_{ii} \quad (13)$$

The above-mentioned method, as known, is based on the variation of the natural frequency of the mode of interest $\Delta\omega_k$ for small physical masses, Δm_{ii} , added at some points of the structure, supposing that the mode remain the same of the unchanged structure. The ratio between the modal mass estimated by the previous equation and the biased one:

$$\tilde{m}_k = \frac{1}{\sqrt{G_{ff} R_{11}^{(k)}} \omega_{d_k}} \quad (14)$$

provides the square root of the power spectral density of the input force:

$$\sqrt{G_{ff}} = \frac{m_k}{\tilde{m}_k} \quad (15)$$

Therefore, the values of the input *PSD* for each mode of interest has been evaluated. Besides, the *PSD* behavior could be estimated as follows:

$$G_{ff}(\omega) = \frac{G_{xx}(\omega)}{|H_{11}(\omega)|^2} \quad (16)$$

being $H_{11}(\omega)$ the *FRF* synthesized for M modes:

$$H_{11}(\omega) = \sum_{k=1}^M \frac{\left(\phi_1^{(k)} \right)^2}{m_k [(\omega_{n_k}^2 - \omega^2) + i(2\sigma_k \omega)]} \quad (17)$$

3 Experimental Investigation

The proposed procedure has been checked on two aluminum cantilever beams in a *SIMO* experimental setup with an impulsive excitation. The dimensions of the first beam were $0.414 \times 0.050 \times 0.003$ [m] and the experimental test, carried out in the frequency range of $0 - 256$ [Hz] by using 8192 sampling points over 5 averages, allowed one to measure four time histories corresponding to the vertical accelerations of the points equally distributed along the beam span. The natural frequencies estimated from the *FRF*'s obtained by the procedure mentioned in the theoretical background (*HTM*) and the ones obtained from the PolyMAX approach present in the *LMS* software Test.LAB are reported in Tab. 1, whereas the different estimation for the damping ratios are reported in Tab. 2.

Mode #	f_n^{HTM} [Hz]	f_n^{LMS} [Hz]
1	13.940	13.921
2	87.374	87.341
3	244.550	244.568

Table 1: Natural frequency estimates for the first cantilever beam.

Mode #	ζ_n^{HTM} (%)	ζ_n^{LMS} (%)
1	0.28	0.19
2	0.25	0.28
3	0.51	0.45

Table 2: Damping ratio estimates for the first cantilever beam.

From these results, it appears that the proposed Output Only method produces the same estimate of the modal parameters as the Input/Output approach. The modal model achieved so far is capable to estimate an accurate frequency response function, especially in the neighborhood of the resonant peaks. This good correlation is reported in Fig. 1, where the first driving point FRF achieved using the H_2 estimator with the Input/Output data together with the one synthesized using the Output Only data is depicted for sake of brevity. The scaling factors needed to unbiased the Output Only FRF have been gained by adding 10 [gr] of lumped mass for each of the experimental degree of freedom.

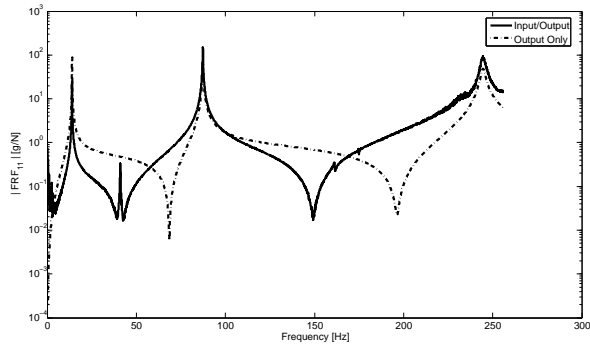


Figure 1: Comparison between the Frequency Response Functions for the first cantilever beam.

Next, the input loading is estimated by applying Eq. 16 described in the previous theoretical section. The result is depicted in Fig. 2 where the actual PSD of the input loading is compared with the one from the proposed approach. Moreover, the partial

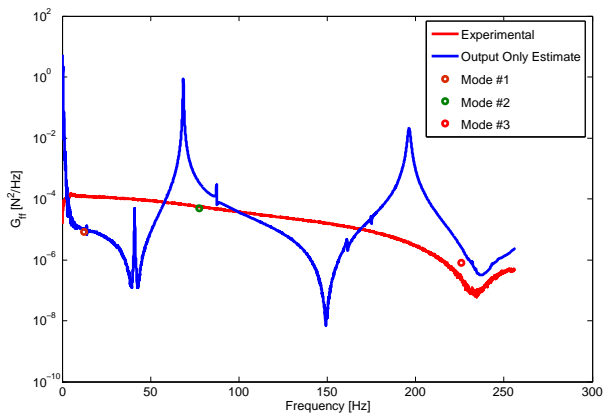


Figure 2: Estimate of the dynamic load for the first cantilever beam.

contribution of each mode to the global spectrum evaluation is evaluated using Eq. 15 and reported in the same Fig. 2. These results have shown an overall acceptable estimation of the unknown input loading except for the poor evaluation of the anti-resonance positions in the FRF estimated by the Output Only approach. This leads to the major differences between the actual and predicted input spectra. Nevertheless, if a narrow frequency band is considered across the poles of the system, a completely acceptable correlation is obtained, as reported in Figs. 3, 4, and 5 for the first, second and third mode respectively.

The dimensions of the second beam were $0.25 \times 0.012 \times 0.002$ [m] and the experimental test, carried out in the frequency range of $0 - 1024$ [Hz] by using 4096 spectral lines over 10 averages with the same type of input excitation, but with a different shape of the spectra, achieved with a different stiffness of the exciting hammer.

Mode #	f_n^{HTM} [Hz]	f_n^{LMS} [Hz]
1	22.779	22.829
2	142.91	142.911
3	403.79	403.819
4	775.75	777.727

Table 3: Natural frequency estimates for the second cantilever beam.

As for the previous experimental test, the modal parameters, estimated by the proposed Output Only procedure, were practically the same as those obtained from the Input/Output method, as one can see from Tabs. 3 and 4. Next, introducing four lumped masses of 2 [gr] each at the experimental grids, the scaling factors could be

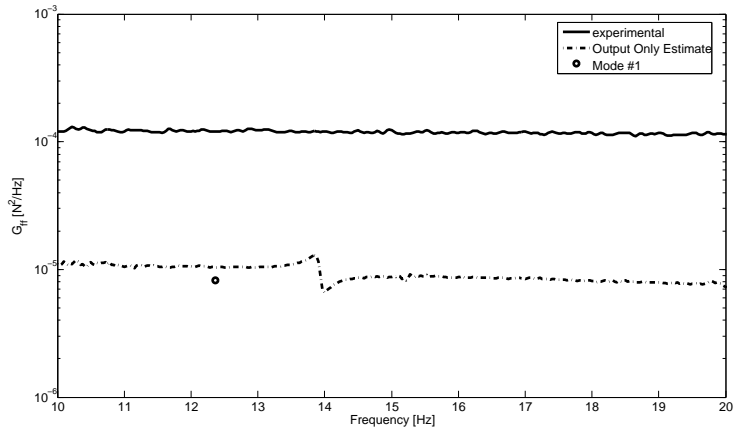


Figure 3: Estimate of the dynamic load for the first cantilever beam - First mode.

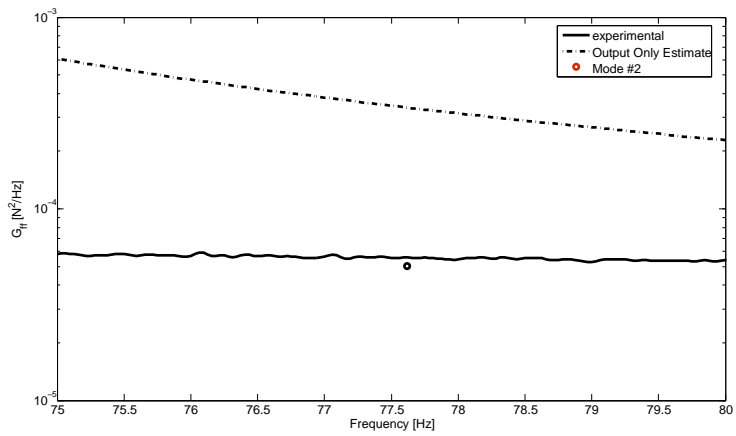


Figure 4: Estimate of the dynamic load for the first cantilever beam - Second mode.

evaluated and then the frequency response functions could be synthesized. Again a good correlation among the FRFs is gained, as demonstrated for the case of $H_{11}(\omega)$ only in Fig. 6.

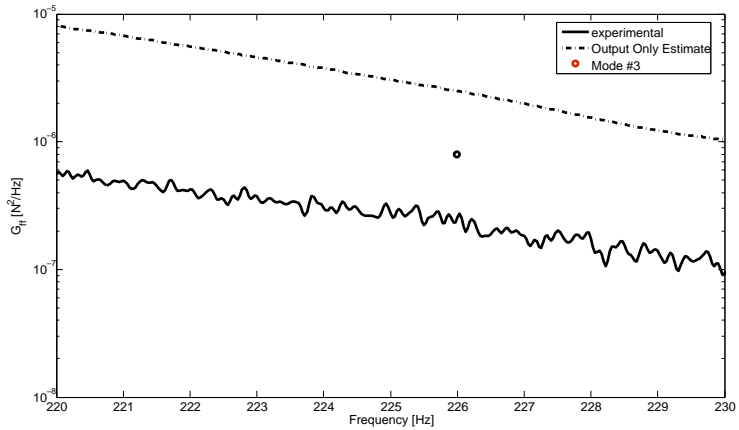


Figure 5: Estimate of the dynamic load for the first cantilever beam - Third mode.

Mode #	ζ^{HTM} (%)	ζ^{LMS} (%)
1	0.12	0.09
2	0.77	0.76
3	0.28	0.20
4	0.38	0.63

Table 4: Damping ratio estimates for the second cantilever beam.

Finally, the prediction of the power spectral density of the input loading is reported in Fig. 7, where it is compared with the actual spectra together with the partial contribution of each mode. Also for this case, the good correlation achieved in the neighborhood of the identified natural frequencies, has an evident loss of accuracy when considering the frequencies across the anti-resonance points. A detailed view of the good correlation around the resonances are reported in Figs. from 8 to 11.

4 Concluding Remarks

In this paper, a methodology suitable for identifying the the Power Spectral Densities of the input forces when the operative responses are only recorded is presented. The approach begins to evaluate the biased *FRF*'s starting from a diagonal term by using the Hilbert transform in order to get the unknown phase and after to derive the out of diagonal functions. So it is possible to estimate the natural frequencies, the damping ratios and the biased residues. From these last ones, the eigenvectors - normalized

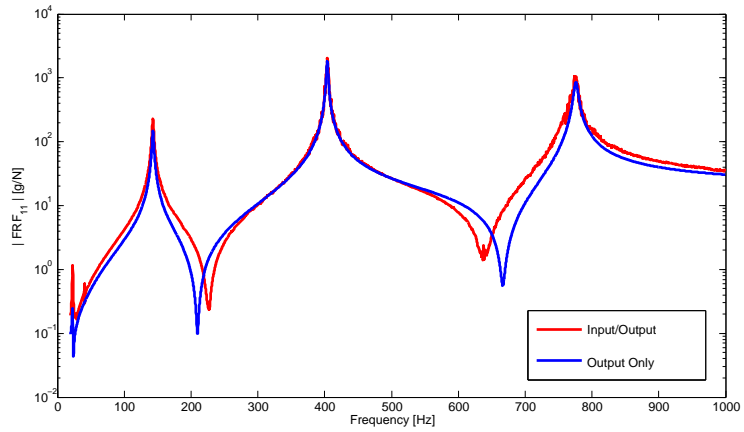


Figure 6: Comparison between the Frequency Response Functions for the second cantilever beam.

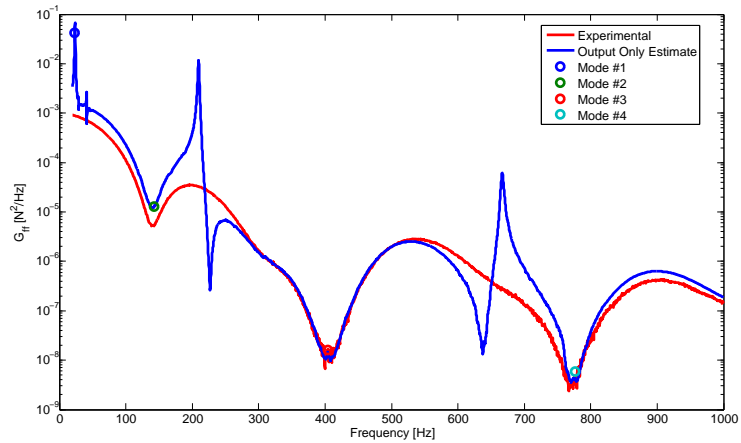


Figure 7: Estimate of the dynamic load for the second cantilever beam.

with respect to one of their component - can be obtained. It is noteworthy that these eigenvectors are unbiased, and they can be used to estimate the modal mass of the mode of interest with the added masses approach along with the synthesized frequency response functions. On the contrary the biased modal masses are gained from the relationship

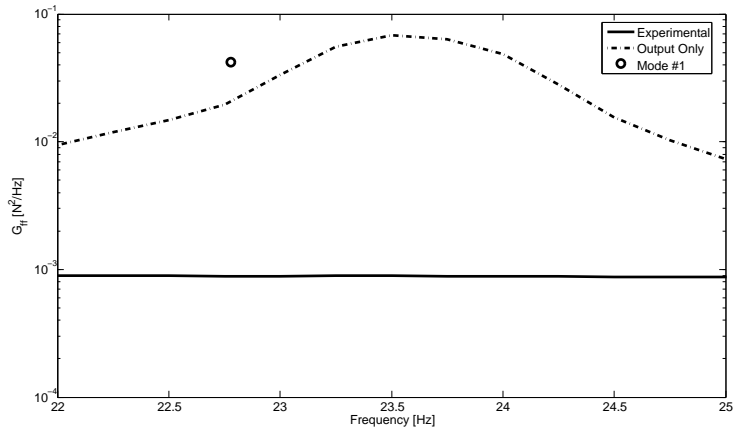


Figure 8: Estimate of the dynamic load for the second cantilever beam - First mode.

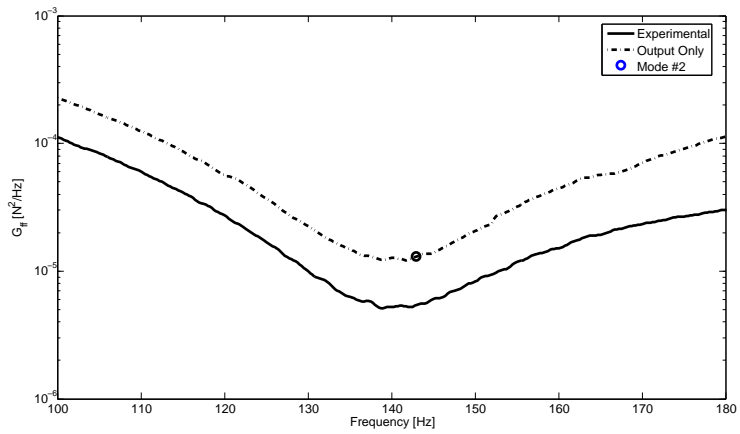


Figure 9: Estimate of the dynamic load for the second cantilever beam - Second mode.

between the biased residue and the eigenvector. Therefore, the value of the input power spectral density, for each resonance considered, can be achieved from the ratio between the modal and the biased modal masses. Besides, the complete *PSD* function could be

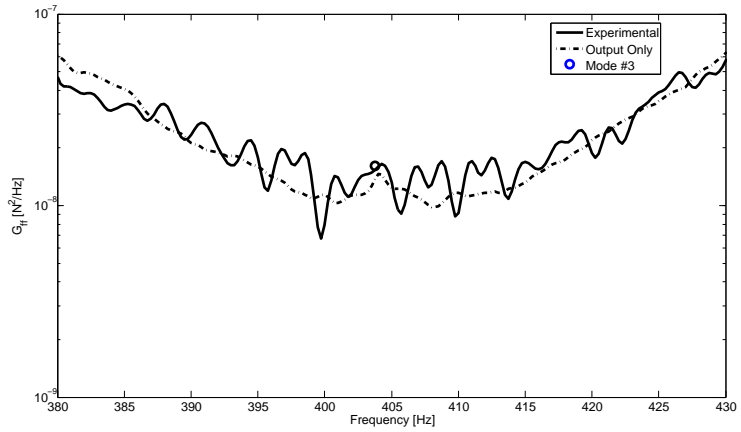


Figure 10: Estimate of the dynamic load for the second cantilever beam - Third mode.

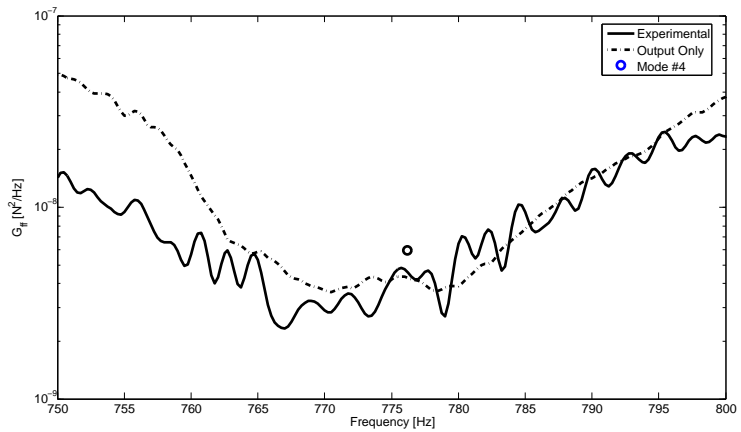


Figure 11: Estimate of the dynamic load for the second cantilever beam - Fourth mode.

derived from the power spectral density of the output and the module of the synthesized frequency response function. In the estimation the errors can be due to the inexact residue estimate at the node considered as a driving point and the eigenvector, derived

from the residues and used to obtain the modal mass. In the second case, the problem, as shown in the paper is represented by the non perfect estimation of the FRF function, especially for the positions of the anti-resonances, in fact this problem gives rise to a notch and peak behavior in the input PSD estimate. In any case, the biased modal masses could provide the possibility to avoid the presence of a control accelerometer when the roving technique is adopted. That will be the aim for a future investigation.

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